AD-A014 998

HELICOPTER FATIGUE LOAD AND LIFE DETERMINATION METHODS

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Daytor University

Prepared for:

Army Air Mobility Research and Development Laboratory

August 1975

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USAAMRDL-TR-75-27

HELICOPTER FATIGUE LOAD AND LIFE DETERMINATION METHODS

University of Dayton Research Institute
300 College Park Avenue
Dayton, Ohio 45469

ADA014998

August 1975

Final Report for Period March 1974 - March 1975

Approved for public release;
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Prepared for

EUSTIS DIRECTORATE
U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
Fort Eustis, Va. 23604

EUSTIS DIRECTORATE POSITION STATEMENT

This report presents the results of an extensive data survey and analysis of the techniques used by the prime helicopter manufacturers in predicting fatigue loads and life of structural components. The principal areas investigated and discussed are mission profiles, flight strain survey, bench fatigue tests, and safe life calculation. Finally, based on the overall findings, a generalized life is presented.

William T. Alexander, Jr. of the Technology Applications Division served as project engineer for this effort.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER USAAMRDL-TR-75-27	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) HELICOPTER FATIGUE LOAD AND LIFE DETERMINATION METHODS		5. TYPE OF REPORT & PERIOD COVERED Final Report-March 1974 - March 1975
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John Patrick Ryan, Alan P. Berens, Richard G. Coy, George J. Roth		8. CONTRACT OR GRANT NUMBER(s) DAAJ02-74-C-0031
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Dayton Research Institute 300 College Park Avenue Dayton, Ohio 45469		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62208A 1F262208AH90 02 007EK
11. CONTROLLING OFFICE NAME AND ADDRESS Eustis Directorate U.S. Army Air Mobility Research & Development Laboratory, Fort Eustis, Virginia		12. REPORT DATE August 1975
		13. NUMBER OF PAGES 73
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

aircraft structures
flight spectra
helicopter operations
flight strain survey
fatigue life
fatigue tests
structural reliability

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Fatigue life estimation of helicopter dynamic components is a complex process that is currently achieved through many different methods. The objective of this study was to review existing methods and to develop a standardized method that could be used by the Army for reliably predicting operational life. This report presents results of a detailed review of fatigue life methods used by five helicopter manufacturers as determined by site

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20. Abstract.

visits and literature reviews. Recommendations for a standardized method are presented in areas of mission spectra definition, flight strain survey techniques, laboratory fatigue strength characterization, and safe-life calculation procedures.

PREFACE

This report, "Helicopter Fatigue Load and Life Determination Methods," was prepared by the University of Dayton Research Institute (UDRI), Dayton, Ohio, for the Eustis Directorate, U.S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia, under Contract DAAJ02-74-C-0031. This effort was conducted between March 1974 and March 1975. The Project Monitors for the Army were Mr. James Waller and Mr. William T. Alexander, who succeeded Mr. Waller early in the program.

This program was conducted by UDRI under the general direction of Mr. Dale H. Whitford, Supervisor of Aerospace Mechanics Research. Program supervision was provided by Mr. Richard G. Coy, Group Leader, Flight Loads. Mr. John P. Ryan was assigned as Project Engineer.

The authors express appreciation to Mr. P. Eodice of Wright-Patterson Air Force Base, Mr. Gordon Woods of the Naval Air Development Center, and industry representatives from Bell Helicopter Company, Boeing-Vertol, Hughes Tool Company, Kaman Aircraft Corporation, Technology Incorporated, and Sikorsky Aircraft for their contributions to this report.

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INTRODUCTION

Structural fatigue life estimation for helicopter dynamic components is a complex process consisting of three basic steps: (1) the estimation of the fatigue strength of the component; (2) the estimation of the magnitudes and frequencies of occurrence of the stresses that will be encountered during the operational life of the structure; and (3) the combination of the interactive effects of these stresses and strengths by means of a model which yields an expected life for the anticipated usage. Over the years the organizations concerned with fatigue life estimation have evolved their own methods of performing the many distinct elements of these three steps. While innovations have been introduced through this individualistic approach, life estimates from different organizations are not comparable and they contain varying degrees of conservativeness. The program described in this report was initiated to lay the foundation for the development of a single Army technique for predicting structural life of helicopter components. The objectives of this program were to analyze the various methods used by Army helicopter manufacturers and to develop a standardized method for reliable life estimation.

The program was performed in two phases, with the first phase entailing an extensive data collection and analysis effort and the second phase entailing the formulation of proposed standardized techniques. At the conclusion of Phase I, reviews were made of the fatigue analysis techniques of each of five helicopter manufacturers as interpreted from meetings with the manufacturers' representatives and available reports. The proposed technique for a standardized method of life estimation comprises the body of this report and is presented in five major sections as depicted in Figure 1. However, before proceeding to the proposed technique, some general comments regarding the basic rationale adopted in this report are presented in the following paragraphs.

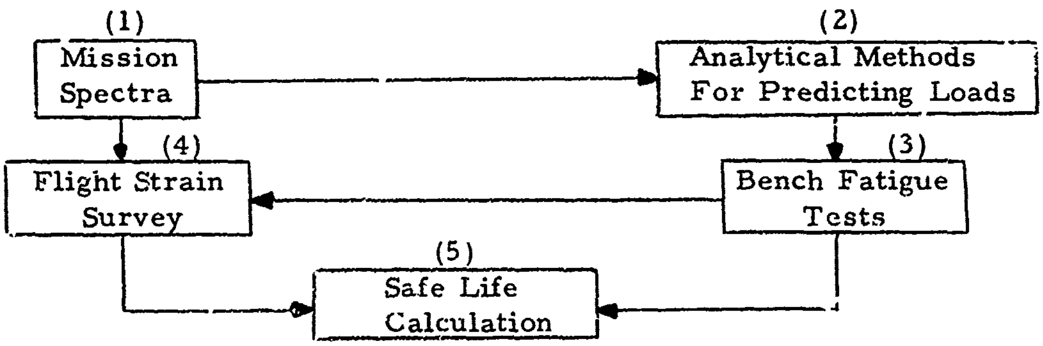

Figure 1. Fatigue Life Methodology.

A basic premise underlying this study is that the structural fatigue life is to be evaluated and not changed. This premise is embodied in the "safe life" design philosophy in which the emphasis is on time to first failure in a fleet. There is no intention here to ignore the "fail safe" design philosophy for which the emphasis is centered on crack propagation rates, the ability to inspect and find cracks before they become unstable, and/or the ability to design crack arrestors or redundancies into the structure. It is also realized that as the scatter in fatigue life increases, the payoff from a "fail safe" design concept also increases. However, the use of the "fail safe" concept does not eliminate the need to evaluate fatigue life during the design phase or to estimate the fatigue life during operational service since the inspection intervals and the extent of acceptable damage are determined by analysis and testing and not by service experience. Further, due to the high rate of occurrence of oscillatory loads in helicopters, cracks grow very rapidly to critical size. Thus, the crack initiation phase, which cannot be modeled by fracture mechanics methods, is the dominant factor in the life length of real helicopter structures.

Given that the objective is to estimate time to failure (or time to crack initiation since crack propagation proceeds very rapidly in helicopters), Miner's theory of damage accumulation is the only viable model that would be acceptable to the entire community. This is not to imply universal acceptance of the ability of Miner's theory to precisely predict fatigue life in an operational environment but rather that no other model is clearly superior when all aspects of the problem are considered. Miner's theory will therefore be assumed as the mechanism for arriving at life estimates, and this assumption has several ramifications.

The input required for a life estimate using Miner's theory consists of the magnitude and frequency of the oscillating stresses and associated mean stresses expected to be encountered by the structure and characterization of the fatigue strength of the structure in terms of S-N diagrams. This input characterization was the major factor influencing the organization of this report. First, the frequency of anticipated significant stresses is considered by means of mission spectra which define expected usage time in flight and ground conditions whose vibratory stresses can contribute to the damage accumulation. Two sections (Steps 2 and 4 of Figure 1) are devoted to the magnitude of the mean and vibratory stresses associated with the stress influencing factors of the flight conditions. The second section presented (Step 2 of Figure 1) is related to the theoretical computations of the stresses. Next is presented a section on laboratory fatigue test methods for characterizing the fatigue strengths. The section representing Step 4 of Figure 1 is related to the flight strain survey techniques used in experimental determination of the loads encountered in the flight conditions. The final section is devoted to safe-life calculation procedures wherein the stresses and strengths are combined.

The analysis of current methods of applying Miner's theory to helicopter fatigue life estimates indicated that gaps (or, at best, unverified assumptions) are still evident in the state of the art. As will be discussed in detail in the ensuing sections, each of these gaps is circumvented by taking a course of action that is judged to be conservative. The result of this series of conservative actions is to arrive at an estimated life that represents a minimum bound on life, but the extent of conservativeness is nondeterminable, even in a probabilistic sense.

Finally, it should be noted that the proposed method consists of an amalgamation of currently used techniques with minor refinements which add somewhat to complexity. The added complexity will be justified on the basis of reducing excessive conservatism and/or providing data which will help to insure that a conservative approach is being used. It will also be apparent that there are several branching points at which there is no real basis for making a choice among alternatives. The more critical of these branch points are prime candidates for future research, and, hence, modifications to the recommended method can be anticipated.

MISSION SPECTRA

The mission spectra used by the manufacturers in their fatigue life calculations show considerable variation in complexity and format. These spectra variations included manufacturers' derived design spectra, modified versions of existing specification data, and interpretation of collected operational data. The choice of spectra was highly dependent on the chronological date when the analysis was performed, the mission types assigned to the helicopter, and the concerned controlling agency.

In order to arrive at recommendations, we define an idealized helicopter mission spectra as:

> "The most representative history of ground and flight conditions that a given vehicle will encounter during its lifetime."

These ground and flight conditions encompass variations and combinations in mission segments, ground or flight modes, airspeed (A/S), gross weight (W), center of gravity (c.g.), external configuration, rotor rates (RPM), engine power or torque, altitude (Alt.) including both in ground effect (IGE) and out of ground effect (OGE), vehicle attitude, loading spectra (including gust and maneuver), stick travel and rates, and any other parameters which affect the lifetime of the helicopter dynamic components. Table 1 presents approaches and recommendations for developing a standardized mission spectra which includes the above considerations. The following paragraphs provide additional background concerning these recommendations and their application to future flight loads investigations.

BASIC CONDITIONS

Basic conditions are defined as the mission segment - operating modes that a helicopter will see in its lifetime. There are numerous mission segment - flight mode combinations that could be used to describe these basic conditions; however, it is imperative that some fixed format be selected soon to standardize the interchange between flight loads investigations and flight strain surveys. Table 2 presents typical combinations of mission segment - operating modes which designate tiered descriptors of the basic conditions defined in Table 3, which was extracted from Reference 1. This format has three major advantages:

1. It is sufficiently general to allow expansion to include all helicopter classes and their unique mission types.

2. It provides a standardized format which is conducive to collecting, processing, and grouping future flight loads data.

TABLE 1 RECOMMENDED MISSION SPECTRA GENERATION		
	ITEM	REQUIREMENTS
1.	Mission Segment - Operating Mode Combinations Defining Basic Conditions	Using Table 2 as a basic format, make necessary changes or additions to cover all steady-state, maneuver and ground conditions to be encountered by the particular helicopter mission type.
	a. Percentage of time for basic conditions	On an interim basis use values shown in Table 3. Reprocess existing flight loads data and analyze all new data using the format above.
2.	Parametric Splits to Basic Conditions Defining Detailed Conditions	Class interval assignments and percentage apportionments must be designated for weight, c.g., altitude, RPM, airspeed and/or load factor (N_Z) as they apply to each basic condition. Independent or joint probability distributions may be used with Army approval.
	a. Steady State	Typical examples are shown in Figure 2 and Equations 1 and 2.
	b. Maneuver	See Figure 2 and discussion on recommended maneuver split generation.

TABLE 2. MISSION SEGMENT - OPERATING MODES

OPERATING MODE \ MISSION SEGMENT	1 Ground Operations	2 T.O./Land/Low Speed	3 Ascent	4 Fwd. Flight	5 Descent	6 Autorotation
Startup	A					
Shutdown	B					
Ground Run	C					
Taxi	D					
Vert. Lift-off		A				
Rolling T.O.		B				
Vert. Landing		C				
Slide-on Landing		D				E
Hover		E				
Lat. Reversals		F		C		
Long. Reversals		F		C		
Directional Reversals		F		C		
Turns (R&L)		G, O	B	B	C	C
Pop-ups		H				
Sideward (R&L)		I		H		
Rearward		J				
Steady-State Fwd.		K	A	A		B
Flare		L				E
Vert. Climb		M				
Vert. Descent		N				
Pushovers						
Cyclic			C	E		
Collective						
Pull-ups						
Cyclic				D	D	
Collective						
Deceleration				F		
Acceleration				G		
Dives					B	
Partial-Power Desc.					A	
Auto Entry						A
Power Recovery						D
Hoist		P				
Special Turns					E	

TABLE 3. STANDARD MISSION PROFILE

Condition	Helicopter Type (percentage of occurrences)					
	Observation	Utility	Utility/ Assault	Attack	Crane	Transport
1. GROUND OPERATIONS						
A. Startup	0.50	0.50	0.50	0.50	0.50	0.50
B. Shutdown	0.50	0.50	0.50	0.50	0.50	0.50
C. Ground run	2.00	2.00	2.00	2.00	2.00	2.00
D. Taxi (if aircraft has no wheels, transfer to 2.X)	1.00	1.00	1.00	1.00	1.00	1.00
	4.00	4.00	4.00	4.00	4.00	4.00
2. TAKEOFF/LANDING/LOW SPEED FLIGHT (<40 knots)						
A. Vertical lift-off (includes transition to 40 knots)	2.66	1.58	0.43	0.22	0.51	0.66
B. Rolling Takeoff (if aircraft has no wheels, add to 2.A)	0	0	0.43	0.27	1.24	1.53
C. Vertical landing	2.66	1.58	0.43	0.22	0.51	0.66
D. Slide-on landing (if aircraft has no wheels, add 90 percent to 2.C)	0	0	0.43	0.27	1.24	1.53
E. Hover (steady)	0.33	2.81	1.61	0.69	6.18	4.14
F. Hover control reversals	0.45	0.31	0.78	0.25	0.97	0.43
G. Hover turns	0.45	0.31	1.04	0.25	1.49	1.22
H. Pop-ups	0.71	0	0.34	0.48	0.53	0
I. Sideward flight	0.45	0.31	0.70	0.25	0.87	0.43
J. Rearward flight	0.23	0.16	0.39	0.13	0.61	0.23
K. Low-speed forward flight (air taxi)	1.29	2.78	1.50	2.77	1.12	4.13
L. Flare	2.67	1.50	0.78	0.48	0.53	2.45
M. Vertical climb	0.33	0.42	0.22	0.69	2.24	0.29
N. Vertical descent	0.33	0.42	0.22	0.69	2.24	0.29
O. Low-speed turns	4.09	0.74	0.87	0.87	0.61	1.22
	16.65	12.92	10.17	8.53	20.89	19.21
3. ASCENT (>40 knots)						
A. Steady-state climb	8.36	9.00	10.35	3.66	9.91	9.83
B. Turns	1.45	2.32	4.65	4.34	0.65	0.99
C. Pushovers	0.34	0.34	1.23	1.16	0	0
	10.18	11.66	16.23	9.16	10.56	10.82
4. FORWARD FLIGHT (>40 knots)						
A. Level flight	38.08	39.02	30.13	33.40	47.58	46.28
B. Turns	8.14	9.91	10.26	11.65	3.02	3.43
C. Control reversals	1.50	1.27	1.13	0.63	0.10	1.51
D. Pull-ups	2.04	0.25	2.54	2.85	0.05	0.10
E. Pushovers	2.04	0.03	2.54	2.85	0	0.01
F. Deceleration	2.04	2.55	2.54	2.85	0.30	0
G. Acceleration	2.04	2.55	2.54	2.85	0.30	0
H. Yawed flight	0.38	0.34	0.44	0.51	0.08	0.10
	56.26	55.92	52.12	57.59	51.43	51.43
5. DESCENT (power on, >40 knots)						
A. Partial-power descent (steady descent)	2.07	5.26	3.18	2.21	9.10	7.79
B. Dive (power on)	3.37	3.50	5.09	3.45	0.05	2.12
C. Turns	3.30	3.31	4.65	9.27	0.61	1.09
D. Pull-ups	0.84	0.10	1.23	2.46	0.03	0.03
	9.58	12.17	14.15	17.39	9.79	11.21
6. AUTOROTATION (power off)						
A. Entries (includes power chops)	0.39	0.39	0.39	0.39	0.39	0.39
B. Steady descent	1.63	1.63	1.63	1.63	1.63	1.63
C. Turns	0.55	0.55	0.55	0.55	0.55	0.55
D. Power recovery	0.24	0.24	0.24	0.24	0.24	0.24
E. Flare and landing	0.52	0.52	0.52	0.52	0.52	0.52
	3.33	3.33	3.33	3.33	3.33	3.33

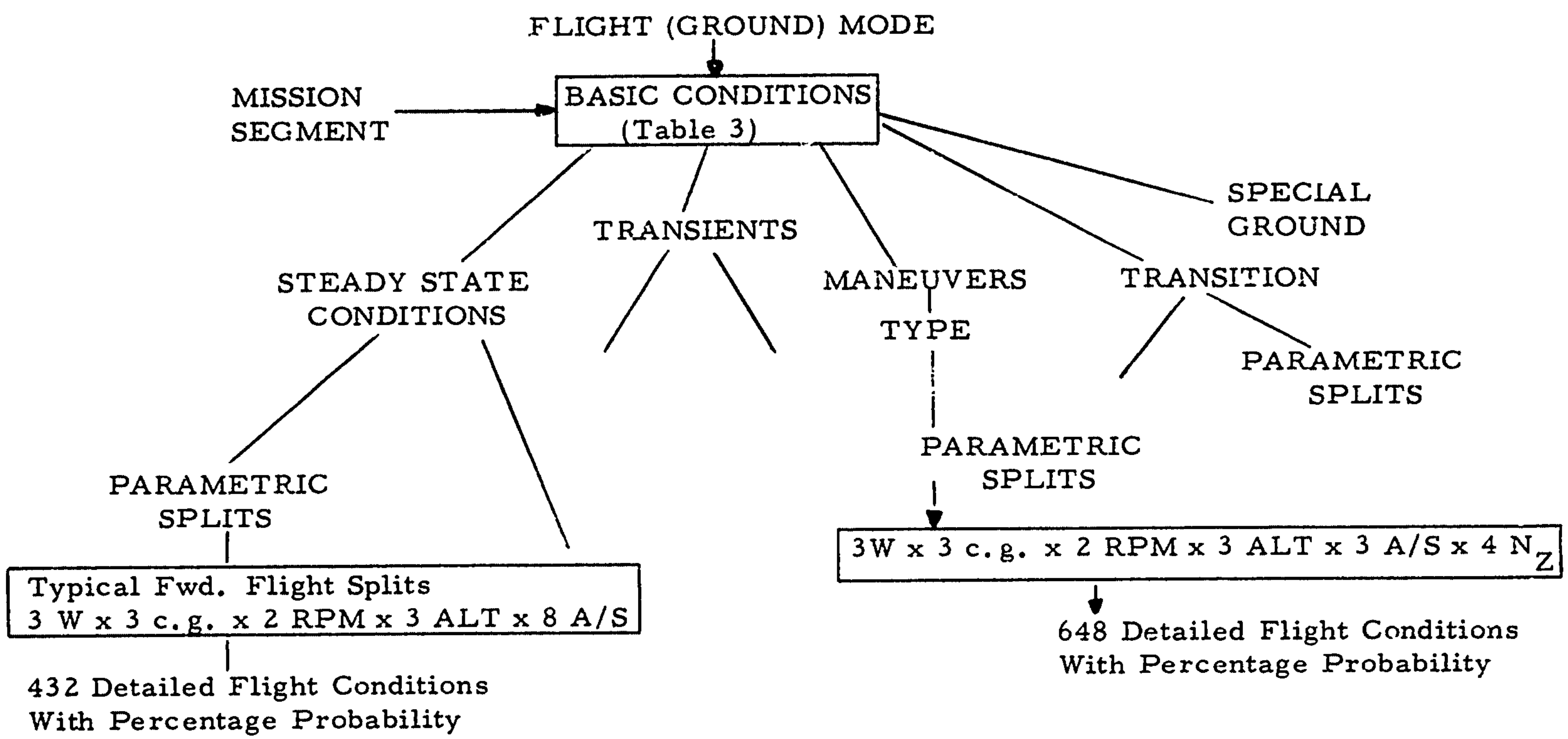

Figure 2. Example Split Application to Basic Conditions.

3. It provides better correlation between flight loads and flight strain survey basic conditions.

The percentage times assigned to the various helicopter classes (Table 3) were derived from four-mission-segment data without benefit of mode breakdown. As a consequence, numerous arbitrary decisions were required in the derivation which can only be confirmed from new or re-analyzed flight loads results. Helicopter manufacturers may negotiate with the Army concerning desired alterations in the percentages shown and/or adding new basic conditions.

DETAILED FLIGHT CONDITIONS

The detailed flight conditions are determined by applying proper distributions of parametric combinations (e.g., airspeed, weight, center of gravity, RPM, etc.) to the basic conditions. The range and class intervals for each parameter and the percentage of time apportionment at these class intervals have often in the past been left to the discretion of the manufacturer. It would seem advisable that the operational loads reports be utilized to assist in defining weight, altitude, RPM, and A/S distributions. These distributions are presently in four-mission-segment format, and thus engineering judgements will be required for application. It will also be necessary to convert these parameter amplitudes to some normalized format for application to new designs. The resultant apportionments will be designated parameteric splits and are often expressed as independent distributions; however, it is likely that joint probability distributions exist between certain parametric combinations and/or levels. In order to simplify the following discussion, independence will be assumed between these parametric splits.

PARAMETRIC SPLITS

Figure 2 illustrates typical split combinations for two of the many basic conditions that will require consideration (i.e., the steady-state level flight and for one type maneuver). A more generalized form for the number of splits for one basic condition and the resulting number of detailed flight conditions would be:

$$\begin{pmatrix}\text{Number of}\\ \text{Detailed}\\ \text{Conditions}\end{pmatrix} = a(W) \times b(c.g.) \times c(RPM) \times d(Alt.) \times e(A/S) \times f(N_Z) \qquad (1)$$

where a, b, c, etc. are the number of class intervals assigned to the respective parameters. Typical class intervals and assigned percentage times for the forward level-flight basic condition might be:

	3 weight class intervals $a = 3$	Assigned Percentage Time
a_1	$W < .7$ max design gross weight	$A_1 = 25\%$
a_2	$.7 \leq W < .9$ max design gross weight	$A_2 = 50\%$
a_3	$W \geq .9$ max design gross weight	$A_3 = 25\%$
	3 center of gravity conditions $b = 3$	
b_1	forward c.g.	$B_1 = 20\%$
b_2	nominal c.g.	$B_2 = 60\%$
b_3	aft c.g.	$B_3 = 20\%$
	2 rotor RPM class intervals $c = 2$	
c_1	RPM $\leq .98$ design RPM	$C_1 = 10\%$
c_2	RPM $> .98$ design RPM	$C_2 = 90\%$
	3 altitude class intervals $d = 3$	
d_1	Alt. $<$ 2000 ft	$D_1 = 30\%$
d_2	2000 ft $\leq$ Alt. $<$ 5000 ft	$D_2 = 50\%$
d_3	Alt. $\geq$ 5000 ft	$D_3 = 20\%$
	5 airspeed class intervals $e = 5$	
e_1	$.32 \leq V/V_H < .5$	$E_1 = 12\%$
e_2	$.5 \leq V/V_H < .65$	$E_2 = 35\%$
e_3	$.65 \leq V/V_H < .80$	$E_3 = 36\%$
e_4	$.8 \leq V/V_H < .96$	$E_4 = 12\%$
e_5	$V/V_H \geq .96$	$E_5 = 5\%$
	1 load factor class interval $f = 1$	
f_1	$.8 \leq N_Z \leq 1.2$	$F_1 = 100\%$

This illustration concerns forward level-flight at airspeed greater than 40 knots (basic condition 4A of Table 3). It points out that if the eight airspeed class intervals of Figure 2 are reduced to five, that the potential detailed flight conditions can be reduced from 432 to 270. An example percentage of time for one of these 270 detailed flight conditions would be:

$$\begin{pmatrix}\text{\% Time}\\ \text{Detailed}\\ \text{Condition}\end{pmatrix} = \begin{pmatrix}\text{\% Time}\\ \text{Basic}\\ \text{Cond.}\end{pmatrix}\left(\frac{A_1}{100}\right)\left(\frac{B_1}{100}\right)\left(\frac{C_1}{100}\right)\left(\frac{D_1}{100}\right)\left(\frac{E_5}{100}\right)\left(\frac{F_1}{100}\right) \qquad (2)$$

Appropriate substitutions for values A_1, B_1, C_1, D_1, and E_5 in Equation 2 illustrate the extremely small percentage of time spent in this detailed flight condition even though the percentage of time spent in the basic

condition (4A of Table 3) varies from 30.13% for utility/assault to 47.58% for crane type helicopters.

It is the manufacturer's responsibility to assign sufficient parameters, class intervals, and related splits to cover all potentially damaging detailed flight conditions that the vehicle will encounter. (This could include adding new parameters and/or additional class intervals to the illustrated case.)

Maneuvers

The proper application of splits to maneuvers is exceedingly more difficult than for the steady-state conditions due to the following problem areas:

1. The present operational loads reporting format does not distinguish between various maneuver types.

2. Although the time spent in operational maneuvering is recorded (maneuver mission segment), individual durations or accumulated time for various subsets are not shown in the multivariate expressions of maneuver counts.

3. Indications point to the fact that the average maneuver duration at more severe load levels is much less than for lower N_Z levels.

4. Application of all splits (weight, c.g., etc.) results in extremely small percentage times, especially when the basic condition percentages are small.

Some manufacturers (References 2-4) have attempted to divide the total operational maneuver spectra into maneuver types by using various engineering judgements and then assigned a fixed duration for each maneuver type. In a slightly different approach the study of Reference 1 ignored maneuver type differences and worked directly with the operational flight loads bivariate tables of maneuver spectra N_Z counts vs class intervals of NZ and the advance ratio ($\mu = V/\Omega R$). The N_Z class intervals denote level and incremental range (e.g., 1.2g - 1.3g). N_Z levels were normalized to the design load factor and the μ values converted to percent V_H using a fixed rotor speed. More details are available in the Reference 1 report; however, two of the conclusions reached were (a) that the percentage of load factor peaks in a given N_Z class interval did not vary significantly with airspeed for a given helicopter type, and (b) that the percentage of load factor peaks in each N_Z class interval could be used as an independent distribution in conjunction with weight and altitude splits based on total operational data.

Based on these conclusions, the Reference 1 study generated plots in normalized form of percentage probability of occurrences in (ΔN_Z - Δ weight) bands and for (ΔN_Z - Δ altitude) bands. This could have been simplified by specifying three independent distributions (one each for N_Z, weight, and altitude) which could be applied as independent splits to the basic conditions using Equation 2. In any event, this development does not answer to the first three problem areas designated above, nor does it account for differences in maneuver time spent at various airspeeds.

Interim Maneuver Split Generation

It is anticipated that future operational flight loads reports will differentiate between types of maneuvers and will designate maneuver spectra in terms of counts and duration at the various load levels and airspeeds. On an interim basis, the following procedural steps are recommended for converting operational data in the present form to usable splits:

1. Collect all maneuver load factor peak data pertinent to the helicopter class. Normalize N_Z load levels for merging data and/or application to new designs.

2. Sum occurrences in each N_Z class interval. It may be necessary to put in cumulative rate form (i.e., exceedances/1000 hours operation), plot the data on semi log graph paper, and extrapolate curve out to at least 1 exceedance per 1000 hours operation. This data can be reconverted to occurrence rate for the desired N_Z class intervals.

3. Determine realistic average time intervals as a function of N_Z class intervals. Take product of occurrences and duration at each N_Z class interval mid range. The probability of peak occurrences in each N_Z class interval is determined as the ratio of each product to the sum of these products over the total N_Z range.

4. Use maneuver mission segment time distributions for weight, airspeed, altitude and RPM split assignments. NOTE: Where maneuvering airspeed time splits are not given, fairly accurate values are derivable by ratioing counts in the 1.2 to 1.3g class interval over any desired airspeed range to the total registered counts in the 1.2 to 1.3g band.

5. The percentages obtained from Steps 3 and 4 may be used in conjunction with the basic condition assignment and Equation 2 to determine the percentage of time in detailed conditions.

This interim procedure does not account for maneuver type differences; however, the duration differences are accounted for.

A typical example of the procedural steps discussed above can be observed from tabulated helicopter transport load factor peaks obtained from References 5 and 6.

1. Tabulated operational maneuver load factor data representative of 165.8 and 235.77 flight hours was extracted from References 5 and 6, respectively. Since this data is from the same helicopter model, normalizing of parameters need not be performed prior to merging the data.

2. Table 4 shows the summed occurrences of load factor peaks from the two data sources of Item 1. In Table 4 there are four N_Z peaks recorded in the N_Z class interval 1.6g to 1.7g for the 401.57 hours of recorded operational data and no recorded peaks greater than 1.7g. Table 5 shows the procedural steps for converting the positive N_Z counts to exceedances/401.57 hours to extrapolated exceedances/1000 hours, to expected occurrence rate per 1000 hours operation, to percentage probability of occurrence. Features of the Table 5 examples are: (a) use of an exceedance rate plot on semi log paper (Step 3) to extrapolate exceedance rates beyond the 1.6g level, and (b) if expected N_Z counts are desired for 4000 hours of operation this procedure must be repeated and extrapolated out to at least 1 exceedance per 4000 hours.

3. Indications point to the fact that the average maneuver duration varies inversely with the load level. The percentage probabilities shown in Step 5 of Table 5 are based on fixed duration at all load levels. Table 6 shows an example calculation for maneuver duration correction. NOTE: Percent probability in class interval (1.3g to 1.4g) = (7353/83,742.9) 100 = 8.78%.

4. There are instances where the airspeed distributions for the maneuvering mission segment are not available. As an interim measure this may be accomplished by utilizing the bivariate maneuver load factor tables. The advance ratio (μ) can be converted to $\%V_H$ based on rotor speed (RPM) and the design V_H. The percentage time spent in each class interval (e.g., $.1 \leq \mu < .15$) should correlate with the ratio of counts obtained in the 1.2g to 1.3g class interval. Using this illustrated μ band in conjunction with Table 4 the percentage time spent in the 49% V_H to 66% V_H class interval is (871/2694) (100) = 32.33%. This procedure can be repeated for the other μ bands and their related $\%V_H$ class intervals.

TABLE 4 TRANSPORT MANEUVER LOAD FACTOR TABULATION									
N_Z Class Intervals	Advanced Ratio (μ) Class Intervals								Total N_Z Counts
	<0.0	0.0 to .05	.05 to .10	.10 to .15	.15 to .20	.20 to .25	.25 to .30	.30 to .35	
1.7 to 1.8									
1.6 t 1.7				1	1	1	1		4
1.5 to 1.6			2		3	2	1		8
1.4 to 1.5	2	9	1	5	16	13	4		50
1.3 to 1.4	6	28	10	24	82	80	27	1	258
1.2 to 1.3	23	150	82	214	871	1055	293	6	2694
.8 to .7	11	92	53	130	581	856	246	5	1974
.7 to .6	2	11	3	6	47	80	22	1	172
.6 to .5		1	2	1	10	10	4		28
.5 to .4						1		1	3
.4 to .2							1		1
TOTAL	44	291	153	382	1611	2098	599	14	5192

TABLE 5 EXAMPLE EXTRAPOLATION PROCEDURE (POSITIVE MANEUVERS)					
N_Z	Step 1 Counts/ 401.57 hrs	Step 2 Exceedances 401.57 hrs	Step 3 Exceedances 1000 hours	Step 4 Counts 1000 hrs	Step 5 % Probability
1.8 to 1.9			1	1	.0142
1.7 to 1.8			3	2	.0284
1.6 to 1.7	4	4	10	7	.0995
1.5 to 1.6	8	12	30	20	.2843
1.4 to 1.5	50	62	155	125	1.7768
1.3 to 1.4	258	320	800	645	9.1684
1.2 to 1.3	2494	2814	7035	6235	88.6283

TABLE 6 EXAMPLE MANEUVER DURATION CORRECTION				
N_Z	Step 1 Duration sic	Step 2 Counts 1000 hrs	Step 3 1 x 2	Step 4 % Probability
1.8 to 1.9	5.2	1	5.2	.0062
1.7 to 1.8	6.1	2	16.2	.0146
1.6 to 1.7	7.4	7	51.8	.0619
1.5 to 1.6	9.1	20	182.0	.2173
1.4 to 1.5	10.55	125	1318.75	1.5748
1.3 to 1.4	11.40	645	7353.0	8.8704
1.2 to 1.3	12.00	6235	74820.0	89.3448
			*83742.95	

*Represents total time (~ seconds) spent in maneuvering, for 1000 hours of operational flight).

ANALYTICAL METHODS FOR PREDICTING LOADS

Each manufacturer has, over a period of years, developed his own computerized loads calculation programs. These huge programs are generally in modular form and have many application options, such as initial design and sizing, component loading, and flight characteristics. Each system has built-in features unique to the manufacturer's closed-loop analytical methodology which uses numerous iterations. A commonality of all rotor aeroelastic simulations is that they effectively have separate structural dynamic and aerodynamic representations which are coupled together by the solution procedure. Some of the details concerning the modeling of aerodynamic forces, dynamic response, and the overall analytical loads prediction methodologies used by four manufacturers are presented in References 7-10.

All four of these programs were designed to analytically determine loading for steady-state flight conditions; whereas two of these programs were designed to predict transient loads due to maneuvers or uniform gusts. In some instances fairly good correlation has been shown between predicted and measured loading; however, further development of these methods is necessary. This is primarily due to technological limitations with regard to the aerodynamic modeling. Some of the problem areas are rotor wake representations, variable drag, and the fact that airfoil section representation is basically empirical rather than predictive.

Despite the potential problem of correlating predicted and measured results (especially with regard to duplicating higher harmonics), many beneficial results have been derived from the analytical programs. These benefits include design improvements, determination of loads sensitivity to various parameters, and performance estimation.

These computer programs will probably never replace the need for flight strain survey programs; however, where life estimates are required in the preliminary design stages, use must be made of these programs in conjunction with the past experience of the helicopter manufacturer.

LABORATORY FATIGUE STRENGTH CHARACTERIZATION TECHNIQUE

The fatigue strengths of critical components expressed in terms of an S-N diagram are an essential input in the fatigue life evaluation process. It is generally accepted in the helicopter industry that this characterization of fatigue strength should be based on real, full-scale components in order to insure that life estimates are based on the realistic test data, particularly with respect to notch sensitivity factors. Accepting this premise, it is recommended that full-scale bench fatigue tests be performed and the resulting data evaluated in accordance with the requirements of Table 7. These requirements are stated in very general terms so that the following paragraphs form an integral part of these recommendations.

TEST METHODS

The objective of the bench fatigue tests is to determine the number of constant amplitude stress cycles that can be endured by a component before it fails. The critical decisions in making such tests realistic depend primarily on the specific design, past experience, and available test facilities. Therefore, it is recommended that the details of such test plans be defined at the appropriate time by the manufacturers and submitted for approval to the Army prior to the initiation of testing. The following comments are offered as being general areas of concern.

It is extremely important, prior to the performance of full-scale bench fatigue tests, that one be aware of the combined loading apportionment on the system and/or full-scale dynamic components. This means that a proper apportionment must be made between flatwise beam bending, chordwise bending, axial loading (including centrifugal force), torsion, drag, and other loading conditions that may arise. This information is usually available from history and from analytical loads work. Even though one loading type may be sufficiently dominant to replace S as the ordinate of the so-called S-N diagram, the combined loading must still be considered. The anticipated amplitudes are necessary to assign proper steady-state mean stresses and to assist in arriving at proper test alternating load levels. Generally, nothing is gained when fatigue tests result in "run outs" (nonfailure) unless the alternating load level is significantly larger than the endurance limit. It is important to be aware of the combined loading variation over the total loading range.

The manufacturers are aware of potential dynamic component critical points and the primary load input from past experience and analytical work. It is assumed, however, that design modifications have been made to

TABLE 7 RECOMMENDED LABORATORY FATIGUE STRENGTH CHARACTERIZATION	
ITEM	REQUIREMENTS
1. Test Methods	Defined by manufacturers subject to Army concurrence
2. Analysis Method	Transform data points to endurance values through specified curve shape equations
	Determine statistics of transformed stresses
	Determine endurance limit stress for working S-N curve
	Determine working S-N curve from endurance limit and specified shape

improve the fatigue strength of these known trouble spots. It is therefore recommended that emphasis be placed on system tests prior to performing full-scale component tests. All fatigue test components should be made using production type processes and techniques. The system and component tests should be highly instrumented (numerous locations and/or axis orientation) to allow verification of the predominant failure load-type distribution over the component. Static pretest loading should be applied for calibration and fixture assessment for all tests. It is the responsibility of the manufacturer to substantiate the fact that the test loading is reasonably close to flight conditions; thus, choice of actuators, method of load coupling, and use of springs or masses is left to his discretion. The constant amplitude or displacement cycles are monitored by instrument, and crack detection wires are often used to forewarn of impending failure.

Worthwhile information can be obtained by detecting the initial crack and recording the additional cycles required for failure. Other valuable information can be determined by investigating the failure mode (fretting, etc.) and/or by using stress coating and other means to assess stress concentration. Some manufacturers perform tests at constant R ratios (S_{min}/S_{max}). The S-N diagram representation is a function of both the mean and alternating stress.

$$S_{mean} = \frac{S_{max}(1 + R)}{2} \tag{3}$$

$$S_{alt} = \frac{S_{max}(1 - R)}{2} \tag{4}$$

Therefore, the mean stress will be variable for all R values except R = -1, in which case the mean stress is zero and the oscillatory stress equals S_{max}. Mean stress corrections will be required for these types of tests.

Ground-air-ground fatigue-sensitive components (e.g., rotor blade root) may be tested in alternate block loadings, one block representing startup operations and the second block the variation in flight conditions.

ANALYSIS METHODS

The S-N data points obtained from full-scale bench fatigue tests are generally in the region of $10^5 \leq N \leq 10^7$ and distributed over various stress levels. If a sufficiently large full-scale data sample could be obtained with data clustered at a minimum of three different alternating stress levels, it would be relatively simple to statistically characterize the fatigue strength

in terms of S-N-P curves at relatively high confidence levels. However, the number of full-scale tests is limited by time and cost, and total sample size for all stress levels is generally $\leq$ six for new designs. To arrive at a characterization of fatigue strength from this small sample size, it has become standard practice in the helicopter industry to transform the observed full-scale test data points to strength values at a predefined number of endurance limit cycles. The transformed strengths are then used to determine a working S-N curve or a distribution of working S-N curves from which damages are calculated.

While this general procedure has become standard practice, the methods of accomplishing the individual steps have not. Coupon data is generally used to determine curve shapes for transforming the observed data, but this is accomplished both graphically and analytically. Some working curves are assigned by graphically determining an S-N curve of proper shape which is a lower bound on all full-scale test points and, hence, on the distribution of test points transformed to the endurance limit number of cycles. All of the methods can be defended as being acceptable and, in fact, would probably lead to quite similar working S-N diagrams. However, since the objective of this study is to recommend techniques which can be standardized, it is desirable to eliminate subjectivity whenever possible. Therefore, the recommended technique for analyzing the bench fatigue test data will follow the accepted general approach but will specify analytical methods for accomplishing the individual steps.

The data analysis has been divided into four steps, and these are discussed in the following paragraphs.

Transformation of Data Points to Endurance Values

The basic rationale for transforming data points to a predefined number of endurance cycles is as follows. The variation in life length displayed by identical test specimens under identical test conditions can be attributed to minute specimen differences which in turn cause each specimen to have a distinct endurance limit. This framework implies a functional relationship between applied stress conditions (S), number of cycles to failure (N), and endurance limit stress (E, a random variable) which can be expressed as

$$S = f(N, E) \tag{5}$$

Knowledge of the distribution of E for the particular full-scale test item can then be used in conjunction with the deterministic function, f, to define working S-N curves.

Now, E cannot be directly determined. Rather, it is necessary to infer values of E from the tests performed at known stress levels and the

observed number of cycles. To draw this inference about the distribution of E values at the predefined number of cycles requires that the functional relationship of Equation 5 be specified. It should be noted that this mathematical framework is not exactly descriptive of nature and also that there is no simple function which would model all aluminums and steels for different R ratios, stress concentration factors, and modes of failure. It is recommended that a set of specific mathematical equations be established for the standardized methodology so that all manufacturers would transform their S-N data to the endurance limit number of cycles in precisely the same manner. These relations should be determined from an analysis of the current base of S-N data and should also specify the mathematical equations for transforming data of different mean or steady-state stresses to a common reference mean stress. The number of cycles at endurance will also have to be specified. Values between 10^7 and 10^8 are currently in use but not all manufacturers use the same value for the same material.

For illustrative purposes in this report, assume that the functional relation of Equation 5 for transforming bench test data to endurance limit stress is expressed by

$$S = E + \alpha N^{-\gamma} \tag{6}$$

(This equation has been used in the past and is recommended in Reference 7.)

The curve shape of the equation is established by the parameters α and γ. A method of estimating these parameters from coupon data is presented in Reference 7. The method consists of using Equation 6 rewritten in the form

$$\frac{S}{\overline{E}_c} = 1 + \frac{\alpha}{\overline{E}_c} N^{-\gamma} \tag{7}$$

where $\overline{E}_c$ is the average endurance limit stress for the particular coupon data set under consideration. Thus, coupon data can be used to estimate the parameters for compositions of materials, failure mode, and notching. Reference 7 presents values of γ and $\alpha/\overline{E}_c$ and some of these are reproduced in Table 8. Note that neither Equation 6 nor the parameter values of Table 8 are being recommended for the standard methodology. Rather, they are being presented merely for illustrative purposes.

To use Equation 7, whose parameter values are determined from coupon tests, it is assumed that

$$\left(\frac{S}{I}\right)_{\text{full scale}} = \left(\frac{S}{\overline{E}_c}\right)_{\text{coupon}} = 1 + \left(\frac{\alpha}{\overline{E}_c}\right) N^{-\gamma} \tag{8}$$

TABLE 8 CONSTANTS FOR EXAMPLE S-N CURVE SHAPES*

Material	Description	$a/\bar{E}$	γ
2024-T4 aluminum	Rotating beam:		
	Smooth	0.40×10^2	0.30
	Notched	0.65×10^2	0.35
	Tension-tension:		
	Smooth, R=0	0.40×10^2	0.36
2014-T6 aluminum	Rotating beam:		
	Smooth	0.48×10^2	0.30
	Notched	0.74×10^2	0.34
	Tension-tension:		
	Smooth, R=0	0.25×10^2	0.30
7075-T6 aluminum	Rotating beam:		
	Smooth	0.76×10^2	0.40
	Notched	0.51×10^2	0.30
	Tension-tension:		
	Smooth, R=0	1.02×10^2	0.44
AZ80-A and ZK60-A magnesium	Tension-tension: Smooth, R=0	0.23×10^2	0.35
4340 steel	Rotating Beam:		
	Smooth, 140 000 psi UTS	3.50×10^4	1.00
	Smooth, 190 000 psi UTS	2.10×10^4	1.00
	Notched, 140 000 psi UTS	0.92×10^4	0.89
	Notched, 190 000 psi UTS	0.47×10^4	0.85
4130 steel	Tension-tension:		
	Smooth, 140 000 psi UTS	3.50×10^4	1.00
	Notched, 140 000 psi UTS	0.92×10^4	0.89
18-8 stainless steel 1/2 hard	Tension-tension: Smooth, R=0	0.13×10^4	0.74

*Note - All of the preceding data are based on tests ranging from a stress ratio of R=-1 to a stress ratio of R=0. For applications where mean stresses differ considerably, additional test data should be reviewed.

When full-scale test specimen i is tested at stress S_i and it fails in N_i cycles, then the transformed endurance limit or fatigue limit stress is given by:

$$E_i = \frac{S_i}{1 + \left(\frac{\alpha}{E_c}\right) N_i^{-\gamma}} \tag{9}$$

This equation defines the transformed or endurance limit stresses which form the sample for determining the working S-N equation.

Statistics of Transformed Stresses

The transformed stress values at the endurance limit form a random sample, and statistical methods must be employed to characterize these stress values. Since the data for the full-scale test items will generally consist of only a few data points, some type of assumption regarding their distribution is highly desirable. (It should be noted that there are distribution-free statistical methods, but their application with small sample sizes is typically unsatisfactory.)

There are two distributions that are commonly assumed for the endurance limit stresses: the normal and the log normal distributions. Considering the degree of skewness in the endurance limit stresses, a very large sample of data points would be required to choose between these distributions, and such large-scale data sets have never been collected. While it may be possible to determine an analytical technique for standardizing data so that data sets from different test conditions and organizations could be combined, such a study was not discovered during this program. Since there is no firm basis for a choice of distributions and since the normal distribution provides more conservative working curves than the log normal, it is recommended that the distribution of endurance limit stresses be assumed to be normal if this distribution is used to make probability statements. Given the normal distribution assumption, the sample mean and sample standard deviation completely summarize the test endurance value stresses.

Endurance Limit Stress for Working S-N Curve

The objective in the analysis of the endurance limit stresses is to determine a stress value which is representative of the weak elements in the population of test items. This stress value can be used with Equation 6 to generate a working S-N curve which is representative of the weak test items. The problem is to quantify the concept of "weak" item by means of

the statistics of the endurance limit stresses. Let E_p be the value of stress that is exceeded by P percent of all endurance limit stresses of the test items. E_p, of course, is never known exactly, as it must be estimated from a sample of data. Let the uncertainty in this estimate be denoted by $E_{p,\beta}$ which is defined by the statement "there is β percent confidence that $E_{p,\beta}$ is exceeded by at least P percent of the endurance limit stresses." $E_{p,\beta}$ is known as a tolerance limit in the statistical literature.

$E_{p,\beta}$ can be calculated by means of the normal distribution assumption for any values of P and β. If it is assumed that both the mean and standard deviation are being estimated from the data sample, then

$$E_{p,\beta} = \bar{E} - K(P,\beta,n)\,\sigma_E \tag{10}$$

where $K(P, \beta, n)$ is shown in Figure 6 for $\beta = 90$. and various values of P and n. A table of $K(P, \beta, n)$ values is available in Reference 12.

Several comments are in order regarding this figure. First, it should be noted that changes in K with sample size reflect the sampling variations that may be present in the estimates of both the mean and standard deviation of the endurance limit stresses. In using this approach it is not necessary to assume that the standard deviation or the coefficient of variation are known in advance.

Second, it is apparent that the effects of sample size on K to obtain a fixed P is quite significant for the smaller values of n, i.e., a considerable increase in precision is obtained for each sample. For example, suppose it is desired to estimate at a 90% confidence level that value of stress which is exceeded by 95% of the endurance limit stresses of the components. To find this tolerance level from a fatigue test of 3 components, the sample mean must be reduced by 5.3 sample standard deviations. From fatigue tests of 4, 5 and 6 components, the sample mean must be reduced by 3.9, 3.4, and 3.1 sample standard deviations, respectively. Note that these values of K are independent of the actual observed values of $\bar{E}$ and σ_E. The increase in precision is due to the reduction in the sampling variation of the estimates for the larger sample sizes.

Finally, if a fixed value of K is chosen for all fatigue tests regardless of the number of specimens tested, the percentage (P) of the population that exceeds $\bar{E} - K\sigma_E$ decreases with sample size. For example, a relatively common practice in the helicopter industry is to reduce the sample mean by three sample standard deviations regardless of sample size. From Figure 3 it can be seen that for sample sizes of 6, 5, 4 and 3 there is 90% confidence that 94.5, 92, 87.5, and 78%, respectively, of the endurance

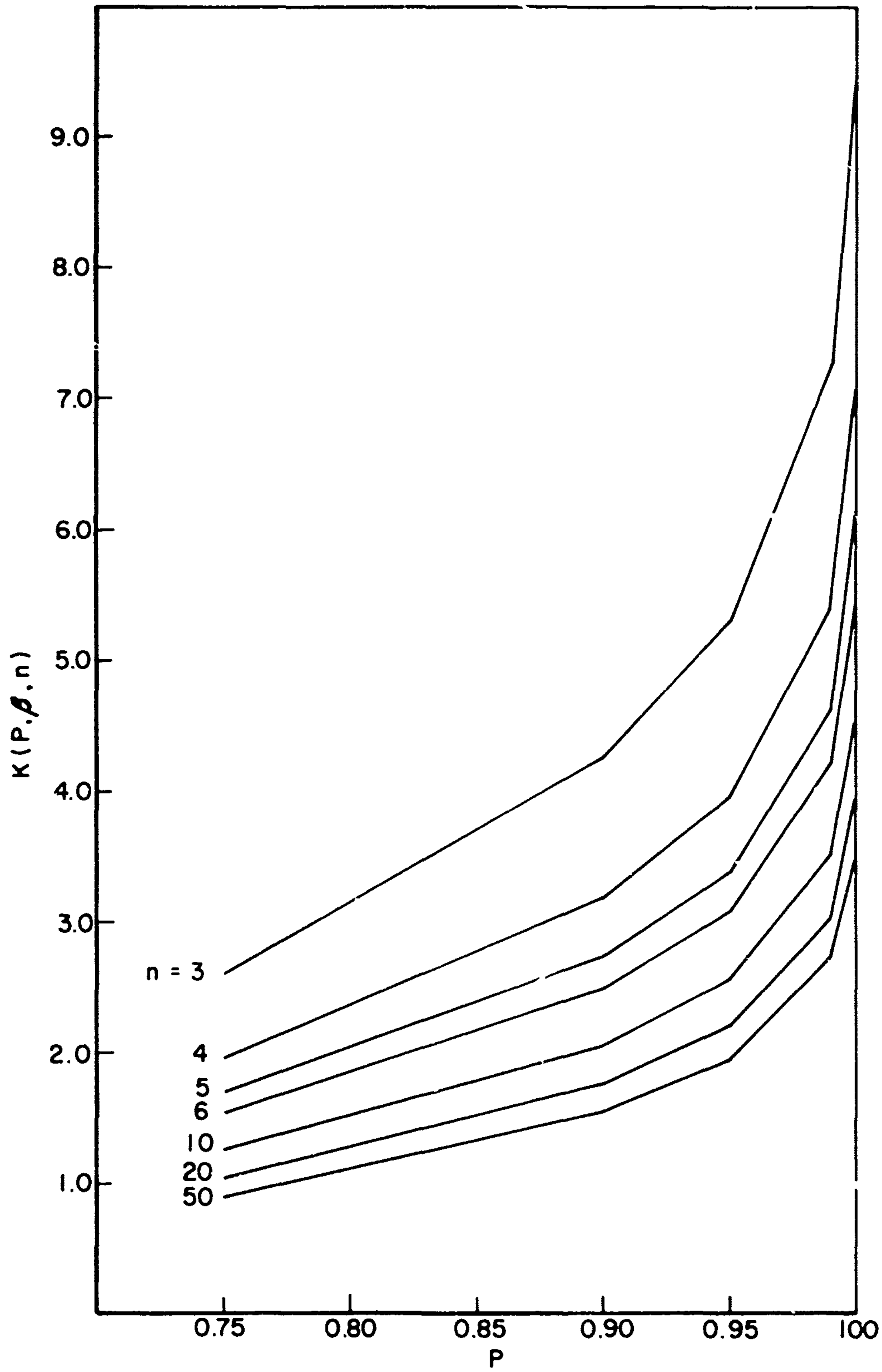

Figure 3. Factors for Obtaining Tolerance Limits for Selected Sample Sizes With 90 Percent Confidence.

limit stresses of the components will exceed $\overline{K} - 3\sigma_E$. Thus, there is wide disparity in the percent of the population covered for this common range of sample sizes. It should also be noted that the "3σ limit" is occasionally interpreted as a 99.87% limit which infers that 99.87% of the endurance limit stresses exceed $\overline{E} - 3\sigma_E$. This is true only if the standard deviation is known, an assumption that is not generally accepted by the industry.

If the observed estimates of $\overline{E}$ and σ_E are used to arrive at a working endurance limit (S_E), it is recommended that the Army specify a level of confidence (β) and the percent of the population of endurance limit stresses to be covered (P). The sample size for any component can then be established through mutual agreement between the Army and the manufacturer. To calculate the working endurance limit, $K(P, \beta, n)$ can be obtained from standard tables (as in Reference 12) and the sample mean endurance limit stress is reduced by K sample standard deviations to arrive at S_E. Note that the sample size should be determined before tests are run and can be different for different components.

Working S-N Curve

The endurance limit of a weak member of the population, S_E, can now be used in conjunction with Equation 6 to determine a working S-N curve for the component. Thus,

$$S_j = S_E + S_E \frac{\alpha}{\overline{E}_c} N_j^{-\gamma} \tag{11}$$

Two comments are in order regarding this equation. First, by choosing different values of P in the determination of $E_{P,\beta}$ and using the resulting values of $\overline{E} - K(P, \beta, n)\,\sigma_E$, S-N-P curves can be generated for the test item. The interpretation of one of these curves is that there is β% confidence that at least P percent of the S-N curves lie above that determined from $E_{P,\beta}$. These probability statements, of course, are subject to the assumptions regarding curve shape and distribution of endurance limit stresses.

Second, the coupon data that will be used to generate values of $\alpha/\overline{E}_C$ and γ may be for certain dominant stress ratios or may be from data converted to a single stress ratio. The S-N of Equation 11 may also have to be modified to account for the effects of steady stresses that are different from those of the coupon tests.

FLIGHT STRAIN SURVEY TECHNIQUES

The flight strain survey tests are performed to provide more realistic replacement data for the analytically predicted loading conditions. The MISSION SPECTRA section pointed out the tremendous number of potentially damaging conditions that must be assessed and, because of time and cost, the need for reducing the test combinations. Any reduction in testing scope must be attended with sufficient proof that untested conditions are nondamaging. In order to apply this reduction, it is necessary to continuously update analytical and statistical results during the testing period. This can be accomplished by judicious use of: loading trends (initially based on analytical studies and past experience followed by trends based on flight test results), full-scale bench fatigue test results, and a properly sequenced flight strain survey. The flight strain survey results are thus used to:

(1) Verify (or disprove) predicted loading sensitivity.

(2) Reduce the scope of detailed flight test conditions by verifying nondamage and/or by substituting more severe loading conditions obtained from a similar environment.

(3) Pinpoint the more damage sensitive conditions to give insight for test modification (e.g., duplicate tests).

Table 9 presents proposed techniques and/or considerations necessary in accomplishing these results. The following paragraphs provide background data for the table as well as areas of concern.

INSTRUMENTATION

Full-scale system and component bench fatigue tests have already been performed; thus, use is made of the pertinent critical point instrumentation and locations as tested. Generally, the flight test instrumentation gives direct recording application; however, in some instances (due to accessability or for multiple application) theoretical relationships are developed to convert measured data to loading at another location. The flight test model(s) must be highly instrumented, not only to give multiple loading measurements for known critical points, but also to monitor airframe loads. The generally considered primary parameters are airspeed, altitude, temperature, RPM, fuel totalizer, accelerations, stick positions, flap angle, etc. Other parameters which should be considered are, for example, vehicle attitudes, attitude rates, and attitude accelerations. In most instances the data is telemetered in digital or FM mode to permanent files

TABLE 9 RECOMMENDED FLIGHT STRAIN SURVEY TECHNIQUES

Item	Requirements
1. Instrumentation	
a. Primary	Strain gages and bridges located per bench fatigue test results. Airspeed, altitude, RPM, torque, temperature, accelerometer, flap angle, trim and control settings, others ar needed.
b. Secondary	Strain gages as required on airframe, noncritical components, attitudes, attitude rates, attitude accelerations, others as needed.
2. Flight Strain Coverage	
a. Pilot Instructions	Sequence, special basic conditions, stick positions, entry and exit rates, N_Z levels to max, trim, record time, duplicate records.
b. Ground	Startup, shut down, taxi, ground run, RPM sweeps, cyclic stick, other parameters as required.
c. Steady State	Climb, descent (dive, partial power, autorotation), sideward, rearward, yawed and forward speed sweeps. Hover, autorotation, hoist, vertical climb and descent. Parametric coverage (Wts, c.g.'s, etc.) as required.
d. Transition	Accelerate, decelerate, takeoff (vertical, rolling) power to auto, auto to power, vertical (climb, descent) flare, land (vertical, slide on). Parametric coverage as required.
e. Maneuvers	
N_Z Related	Pull-ups and pushovers (cyclic, collective), turns (right and left), popups. Parametric coverage as required including N_Z.
Other	Control reversals (lat., long., directional). Parametric coverage as required including stick deflection.
3. Evaluation and Treatment of Flight Strain Data	
Data Reading	Give proper signatures to each record (i.e., test flt. no., record no., basic cond., Wt., c.g., RPM, A/S, other).
Steady-State	Use mid five seconds of each loading record (truncate ends). Read primary vibratory peaks and ratio of secondary tertiary, etc., to primary load for each revolution. Determine mean or steady-state load each record.
Maneuvers and Transients	Manually edit records from time duration. Correlate with N_Z and stick traces. Read peaks and ratios as above and determine mean (st. state) at each primary cycle.
Preliminary Assessment	Determine benign flight conditions (each critical point) based on statistical analysis and loads sensitivity. Define conditions requiring duplicate records.
Finalized Assessment Including Special Considerations	Generate statistical loading distribution of primary cycle data by detail condition and critical point. Obtain factors for secondary, tertiary, etc., cycles.

at the test center, where the flight test engineer has simultaneous quick-look capability as well as direct contact with the pilot.

FLIGHT STRAIN SURVEY COVERAGE

The stress or loading history environment of the dynamic components must be assessed for potential damage over the multitude of detailed flight conditions that the helicopter will see in its lifetime. These loading histories are obtained from actual test flights which compose the flight strain survey.

The test flights are preplanned (including pilot instructions), and the direct contact link is used primarily for minor modifications to the program and for safety purposes. The flights are gross weight-configuration-center of gravity dedicated. Planned fuel sequencing and burnoff allows the tests to be performed within defined tolerances of gross weight and center of gravity. Short recordings are taken (approximately six seconds for steady state and for the maneuver durations) in the designated sequence.

The detailed flight strain survey testing procedures and sequences are the responsibility of the manufacturer. However, with the need for duplicate records and the possible consideration of thousands of detailed flight test conditions (see Figure 2) it would appear beneficial that the more potentially damaging conditions be flown early in the program. It is also important to analytically reduce the potential test combinations by merging conditions and by formulating joint probability groupings. Generally, the greatest impetus for consolidation of test condition combinations occurs after verification of benign damaging conditions. Typical examples of coverage by manufactuers in flight strain survey tests are shown in Table 10. This table points out instances where use was made of joint or combined groupings as well as the application of independent parametric splits such as shown in Equation 1. Joint or combined parameter groupings and/or data arrays with empty cells (described in the comments column) reduce the number of detailed flight conditions assigned to each basic condition. Independent parametric split application (full data arrays) to a basic condition, results in the number of detailed flight conditions equal to the product of the Table 10 values for a, b, c, ... In general the number of detailed conditions within a basic condition that were flight tested were considerably less than the Figure 2 illustration. Furthermore, the number of basic conditions that were considered were much less than shown in Table 3. It is assumed that justification was found for reducing these test parametric levels and basic conditions. Possible reasons for the test scope reduction could be that the most severe parameter levels were tested, that little variation in loading occurred with some parameters, and/or that they considered that subsequent data treatment (such as merging) would provide an offsetting conservatism.

TABLE 10 TYPICAL PARAMETRIC CONSIDERATIONS

Helicopter Type	Basic Condition Type	W	C.G.	ALT	RPM	A/S	N_Z	Comment	Detailed Cond.
Utility	Steady-State	4	1-2	1-3	2-4	8	-	*36 W-c.g.-RPM-alt	288
	Maneuver	4	1-2	1-3	1-2	1-3	?	14 W-c.g.-RPM-alt	42
Transport	Steady-State	3	1-2	1-3	2	6	-	**14 W-c.g.-RPM-alt-	84
	Maneuver	3	1-2	1-3	2	1-2	1	configuration	14-28
Observation	Steady-State	1	1	1-3	2	7	-	2 stick	14-42
	Maneuver	1	1	1	2	1-3	2	rates	4-12
Utility	Steady-State	4	3	1	2	9	-	Maneuver c g.	216
	Maneuver	4	?	1	1	4	10	undefined	160
Crane	Steady-State	3	-	4	-	1-5	-	Most severe loading	12-60
	Maneuver	1-3	-	2-4	-	1-5	8	assigned	9-28

NOTE: Numbers denote parametric class intervals and parametric test combinations.

* Data arrays with empty cells.

** Joint or combined parametric grouping.

In outlining the flight strain survey tests, emphasis must be placed on pilot instructions. This includes not only the outlined sequence of basic conditions and related parametric combinations, but also entry and exit rates, desired N_Z levels, asymmetrical maneuvers, ascent and descent rates, power settings, turning rates, record length, number of duplicate recordings, and any other special instructions indigenous to the helicopter capability and intended usage. As mentioned before, a considerable amount of interdependence exists between the flight strain coverage and the preliminary evaluation and treatment of the data. The coverage shown in Table 9 (Item 2) serves only to point out the conditions which must be investigated. This does not mean that tests must be performed at all these conditions; however, justification must be given whenever substitutions, deletions or other test scope reductions are introduced. The following paragraphs present a method whereby a reduction in test scope and a statistical representation of loading can be accomplished.

EVALUATION AND TREATMENT OF FLIGHT STRAIN DATA

In general there are two levels of sophistication used by the manufacturers in processing and evaluating the helicopter flight strain survey records. The preliminary assessment serves as an indicator of benign or nondamaging conditions, the questionable damaging conditions, and the most probable damaging conditions. In this preliminary damage assessment phase a greater degree of confidence can be obtained for the damage classification by increasing the number of duplicate flight test records and/or increasing the data sample by combining similar test conditions in conjunction with loading trend corrections. The more sophisticated analysis is generally utilized for the conditions which incur appreciable damage. Quite often these more damage sensitive conditions show extreme load variation in the flight test record, and the damage analysis tends to be less conservative.

Data Reading

It is assumed that each flight record is or can be converted to digital format. Proper identifiers can be assigned to these records, such as flight test number, recorded sequence, basic conditions, weight, c.g., ALT, RPM, A/S, and other designators. Automated processing can be utilized to determine the data arrays for each critical point. Pertinent data for steady-state conditions are as follows:

1. Primary peak oscillatory load for each rotor revolution (each record is fixed length).

2. Average steady state or mean load per record.

3. Load ratio secondary to primary, tertiary to primary, etc., for each rotor revolution.

The maneuver records may require manual editing to insure that proper record length is recorded. Other modifications required for maneuver records are the generation of arrays of the steady-state load, N_Z level, and/or stick position for each revolution.

Preliminary Assessment

At this stage it is assumed that S-N diagrams are available for each critical point with a defined endurance limit load. This endurance limit is based on a calculated mean or steady-state load condition and some defined reduction of the average S-N curve. All alternating loads less than this endurance limit are assumed to give zero damage (i.e., unlimited cycles). Thus, one needs only prove that the primary load peak cycles are less than the endurance limit to verify nondamaging conditions. The preliminary assessment may be made at various stages during the flight strain survey program. In steady-state tests, each flight strain record defines a critical point peak cycle array consisting of 16 to 40 stress or load cycles (corresponding to rotor revolutions in a five-second time increment). These peak stress cycle arrays may occur at a mean or average stress other than the mean stress used in generating the S-N diagrams. A correction must be made to compensate for this discrepancy through the use of constant life diagrams such as those of Reference 13. This correction is accomplished through the transformation of the working S-N diagram endurance limit S_E) and associated steady-state stress (S_{mean}) to an equivalent flight test related endurance limit stress ($S_{E(FLT.\ TEST)}$) based on the flight test mean or average stress for the given record. Each flight test peak alternating stress ($S_{alt(FLT.\ TEST)}$) can be converted to percentage of endurance limit by:

$$\frac{S_{alt(FLT.\ TEST)}}{S_{E(FLT.\ TEST)}} 100 = \%\ \text{endurance limit} \qquad (12)$$

These transformed peak cycle stress arrays can be presented in histogram form and/or merged into any grouping desired to form plots of percentage occurrence vs percentage endurance limit. A sketch of such a distribution is shown in Figure 4.

Each individual critical point record will have its own distribution and can be merged to form any desired data record grouping. The percentage endurance limit is a measure of the damage potential. Therefore, flight conditions whose data points lie to the left of the dashed line in Figure 4

are probably nondamaging, those in the crosshatched region are questionable, and those at 100% endurance limit or greater are most probably damaging. It is conceivable that the distribution of peaks for two duplicated detailed flight conditions (same basic condition and same parameteric combinations for weight, c.g., etc.) could be quite different (e.g., Cases A_1 and A_2 of Figure 5). On the other hand, one manufacturer stated that their peak cycles and duplicate record distributions were highly repeatable (e.g., Cases B_1 and B_2).

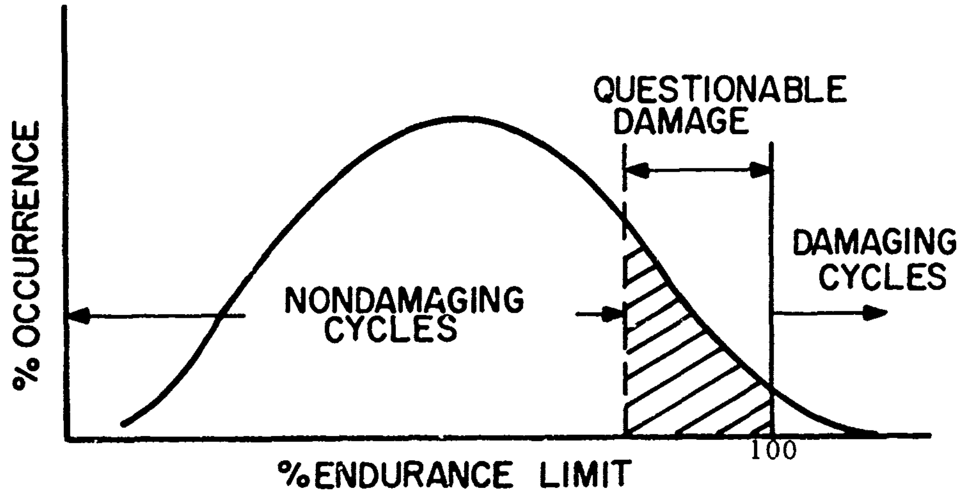

Figure 4. Illustrated Loading Distribution.

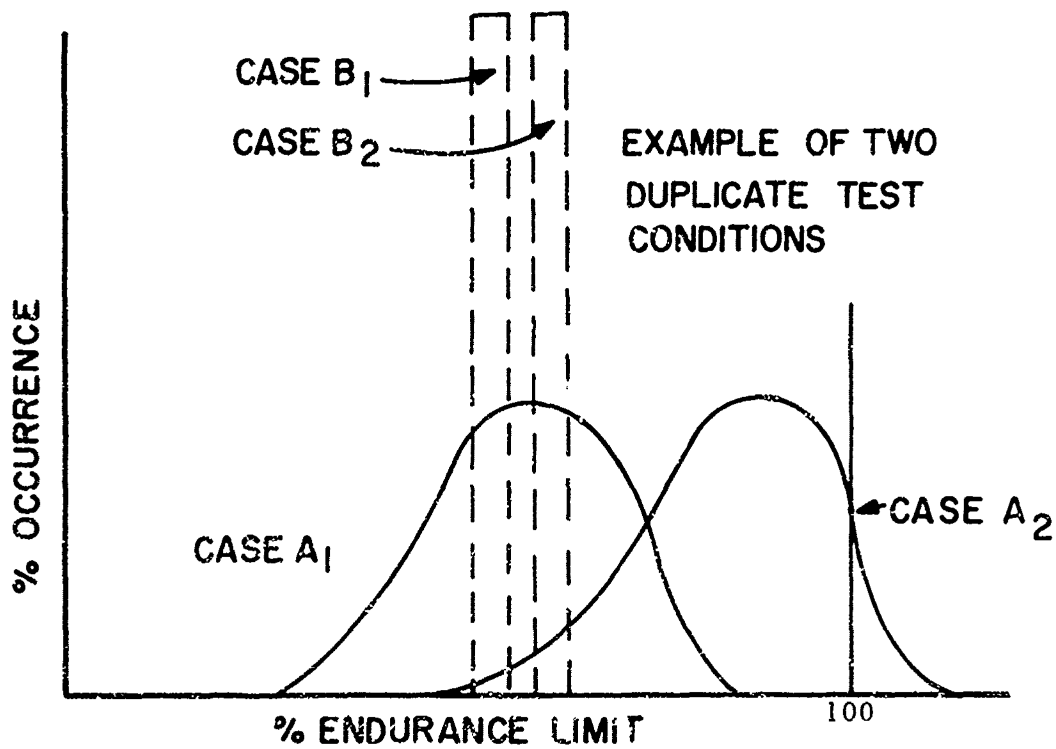

Figure 5. Duplicate Flight Record Distributions.

Cases A and B of Figure 5 illustrate two extreme probability distributions for duplicate tests. Case A points out two important features: first, if distribution A_1 were obtained for the detailed condition on a one-shot basis, then the conclusion could well be that it is nondamaging; if, on the other hand, A2 had been "the only record", then the damage would have been assessed according to the upper load test value. The large shift in the mean between Cases A_1 and A_2 indicates a possible need for a third record. The statistical approach for determining the required number of duplicate tests and/or the resultant statistical reliability is discussed in a later section.

To summarize the preliminary assessment procedures, the following steps are recommended:

1. Perform flight strain survey tests using steady-state level-flight speed sweeps covering most of the parametric levels for weight, c.g., RPM, altitude, etc.

   a. Utilize the recorded peak distributions to determine load trends as a function of the parameters and levels.

   b. Merge records of nonsensitive parameters to increase sample size and give greater confidence in defining non-damaging conditions.

   c. Designate duplicate flight tests for flight conditions found to be potentially damaging.

2. Obtain flight test records as required for the remaining steady-state and quasi steady-state basic conditions. Flight strain survey records are not essential for each detailed flight condition (i.e., all combinations of parametric levels) provided proof is shown of nondamage.

   a. Use results of Step 1 to reduce testing scope (trends and comparison).

   b. Repeat Steps 1b and 1c for this data as required.

3. Obtain transition records for all basic conditions with attendant parametric combinations.

   a. Treat transition data in a similar manner as the steady-state above. Special statistical consideration must be given to the variable record lengths.

4. Obtain maneuver records for all basic conditions and associated parametric combinations.

   a. Plot N_Z versus peak stress for load-factor-related maneuver types and parametric conditions (see Figure 6).

   b. Use plots similar to Figure 6 to determine trends and distributions at various N_Z slices.

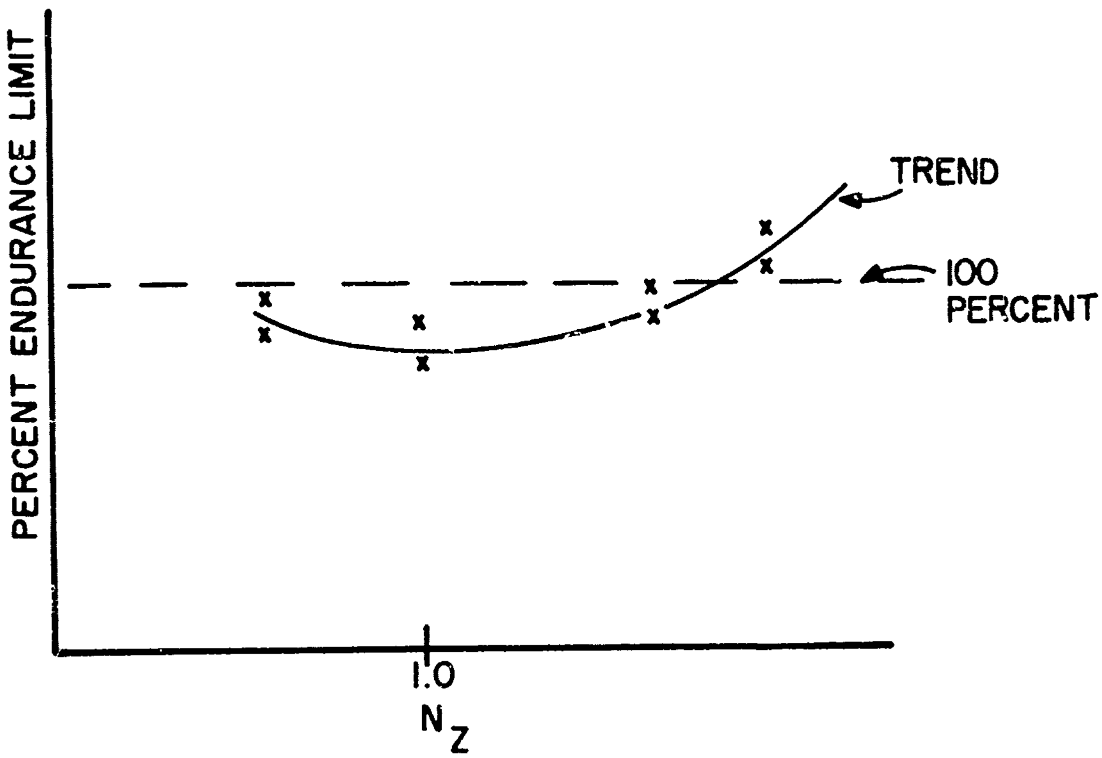

Figure 6. Stress vs Load Level.

   c. Designate nondamaging conditions. Request duplicate records for potential damaging conditions.

   d. Plot stick position vs percent endurance limit for control reversals. Use same procedure as for load level to delineate potential damaging conditions.

The preliminary analytical procedures outlined above appear to be extremely detailed and circuitous; however, the problem complexity is reduced considerably when one recognizes that very few damaging detailed flight conditions exist. It is pointed out that manufacturers spend a considerable amount of analytical effort in an attempt to make all steady-state and quasi steady-state flight conditions nondamaging. The detailed procedures

outlined above are primarily to reduce the possibility of missing any particularly damaging excitation modes. One parameter that requires special attention is the airspeed, which frequently shows reverse curvature loading trends in the low-speed range and has been known to introduce higher order excitation over a narrow airspeed band.

The end products of this preliminary analysis are listed below and will be utilized in the proposed finalized assessment.

1. A list of detailed flight test conditions found to be potentially damaging, each supported by at least one duplicate test record.

2. Tabular arrays of primary peak counts vs percentage of endurance limit for each critical point test record. NOTE: The endurance limit must be modified to account for differences in steady-state stress.

3. Values for mean and standard deviation for each peak alternating stress record.

In most instances the preliminary analysis procedures used by the manufacturers were based on the maximum primary peak for each record. Often the largest primary peak of a merged grouping was considered representative for the time spent in said grouping. For example, using the basic condition 4A of Table 3, the forward level-flight steady-state peaks were grouped in the following manner:

Data Group
A. All peaks by $\%V_H$ grouping.
B. All peaks by $\%V_H$ and altitude grouping.
C. All peaks by $\%V_H$, altitude, and weight grouping.
D. Peaks by detailed flight condition.

In data group A the load variation with respect to weight, c.g., altitude and RPM was ignored and the largest peak cycle of all the grouped records was used to assess the damage. The percentage of time assigned at this $\%V_H$ level would be according to Equation 2 = $E_i/100$ x % time in level flight. If the maximum peak of this grouping was found to be damaging, then the manufacturer had the option of using successive orders of reduction in conservatism (data groups B, C, D). These more refined groupings introduce additional splits as well as reduce the sample size. In some instances (usually maneuvers) the manufacturers concluded that some of the more severe primary peaks were so damaging and/or so unique to the rest of the record that even the detailed flight condition duration (data group D) would be excessive. These special cases were treated in a slightly different

manner according to the manufacturer and will be discussed in the next paragraph. In instances where the primary cycles were found to be damaging, and other load cycles occur in the same rotor revolution, then it becomes necessary to assess the possibility of damage from these secondary, tertiary, etc., cycles.

Existing Finalized Assessment

In the review of five manufacturers' procedures and methodologies, all except one employ some form of cycle count damage assessment. The methodology of this one manufacturer is sufficiently unique to require special consideration be given to their approach. The methods and use of cycle count procedures are as follows:

Manufacturer A determines factors and uses these factors for all flight conditions. He determines the peak oscillatory load for each total record which relates to life cycles N*. He also calculates damage of primary cycles (one per revolution) by cycle count method:

$$\text{Cycle count damage} = \sum_{i=1}^{n} \frac{1}{N_i}$$

For a record length of (n) rotor revolutions, the ratio

$$\text{CCF} = \sum_{i=1}^{n} \frac{1}{N_i} \Big/ \frac{n}{N^*} \tag{13}$$

is actually a ratio of damages using two different methods and is called cycle count factor (CCF). According to the manufacturer, most steady-state and maneuver CCF's were found to be normally distributed and to have magnitudes of less than 0.5 in over 90% of the cases. For turns and extreme maneuvers, the CCF's were determined from actual records. These CCF's were used to reduce the equivalent applied cycles in the damage calculations.

Manufacturer B uses cycle count methodology only for conditions which incur significant damage. He calculates the damage from a specific record. This record is the most severe parametric grouping for the given basic condition. The actual recorded counts are put into histogram form according to defined stress increments, and these counts are prorated to 100 hours of operational flight based on percentage of time and RPM. Certain conservatisms are: (a) primary peak is (max-min)/2 in each

revolution, and (b) the upper limit stress of a given class interval is used to determine the life cycles (N_i). The damage per 100 hours operation is the $\Sigma n/N$ for all stress increments based on primary and higher order cycles.

Manufacturer C modifies the S-N diagram(s) to make the correction applicable to all conditions. Cycle count damage rates for various detailed flight tests were converted to damage per 1000 hours operation. In a parallel effort the main rotor blade peak bending moment for each flight test (M_{p_i}) was recorded. The equivalent number of cycles (n_{e_i}) assigned to the detailed flight condition (i) was specified as:

$$n_{e_i} = \frac{\%\text{ time detailed cond.}}{100} \times \text{RPM} \times 1000 \times 60 \qquad (14)$$

An assumed average RPM was assigned. The effective allowable cycles for this condition is

$$N_{e_i} = \frac{n_{e_i}}{\text{damage/1000 hours}} \qquad (15)$$

The equivalent moment (M_{e_i}) is solvable from the working M-N diagram and represents the equivalent peak moment which gives the same damage as that obtained by cycle count. M_{p_i} vs M_{e_i} data points were plotted for numerous detailed conditions and were used to alter the S-N diagram.

Very little was found concerning the methodology used by Manufacturer D in their cycle count corrections. In the reports reviewed, no cycle counting was used; however, mention was made of the possibility of using this methodology.

<u>Recommended Finalized Assessment</u>

The proposed preliminary assessment procedures and results are utilized to define the final loading spectra for the various critical points. The flight test results obtained from the preliminary assessment provide actual cycle count data by critical point in tabular frequency distribution form for each detailed flight test condition found to have one or more primary peak damaging cycles. Each distribution is accompanied by the necessary data and supporting signatures to allow sorting, merging, etc., as required. There is at least one duplicate distribution for each condition, and the number of data points in each will vary from 16 to 40. The frequency distribution format is the number of occurrences vs percentage endurance limit. In addition to this information, there are also some statistical results

obtained from the duplicate records and data for the secondary, tertiary, etc., load ratios with respect to primary. These analyses will be discussed in the following paragraphs as special considerations.

The desired information from the primary peak data is realistic estimates of the peak load distributions for each critical-point detailed flight condition. This data can be obtained from a large number of duplicate flight tests for each condition, but the cost of this approach would be prohibitive. The recommended final assessment of the load peaks (as described later in special considerations) involves assumptions regarding the form of the distribution of peak loads and the parameters of these distributions. To test the validity of these assumptions requires actual test data, and their acceptance would probably require test data from more than one source.

The proposed method of evaluating the load peaks requires multiple flight strain test records for some of the flight conditions (those most likely to be damaging) and permits the elimination of those flight conditions which can be shown to have little likelihood of causing damage. This is accomplished by using the multiple test results to generate various estimates from which confidence limit statements can be established for the loads distributions. Thus, for example, if it can be shown that there is 95% confidence that 99.99% of the loads in a given condition are nondamaging, then this condition could be eliminated from further consideration.

This recommended procedure achieves a known degree of conservatism with relatively few test records (at the expense of an added assumption) by means of the level of confidence and percentile measure of high load. These latter percentages are at the discretion of the Army. This procedure is contrasted with existing methods of assigning the maximum load observed in the tests for a specific flight condition as being representative of the largest load that will be encountered in the flight condition. No quantitative statement of conservativeness can be made for this procedure; in fact, the selected load may not be conservative if the data sample (for some reason) produced low loads. For example, see Case A_1 of Figure 5.

Assume, then, that expected loading distributions for each potentially damaging detailed flight condition have been derived. Furthermore, assume that the relations for the S-N curve shapes for each critical point are available:

$$\frac{S}{S_E} = f(N) \tag{16}$$

or

$$N = g(S/S_E) \tag{17}$$

The loading distribution is essentially in histogram form of percentage occurrence vs percentage endurance limit. Since $S/S_E \times 100$ is also percentage endurance limit, it is very simple to determine the life cycles and percentage of critical condition time spent at each load level.

Special Considerations

Distribution of Load Peaks

Consider the problem of determining the distribution of the primary peaks of the load cycles that will be encountered in a given flight condition. Since a flight condition is defined in terms of ranges of parameters, different excursions into a flight condition will not necessarily duplicate all parameters, and these differences could cause a change in primary peak load cycles. Further, in one excursion (or flight record) in a flight condition, there can be variation between the primary peak values. Thus, it is conceivable that the random variable of primary peak load cycle could be modeled by

$$Y_{ij} = \mu + B_i + \epsilon_{ij} \tag{18}$$

where μ is an overall mean peak primary load for the flight condition, ϵ_{ij} is a random differential effect due to peak-to-peak variations within an excursion into the flight condition, and B_i is a random differential effect due to different excursions into the flight condition.

Now assume that B_i and ϵ_{ij} are uncorrelated, that they are normally distributed with variances of σ_B^2 and σ_ϵ^2, respectively, and that ϵ_{ij} has zero mean. Reasonable estimates of σ_ϵ^2 can be obtained from one or two records since each record contains a reasonably large sample of primary peaks. However, if only two data runs are obtained for the flight conditions, there is only a sample of size 2 with which to estimate σ_B^2. If, however, the additional assumption is made that σ_B^2 is constant for all flight conditions under investigation, then there are statistical techniques available for estimating σ_B^2 from the pooled sets of data. This technique is known as an analysis of variance for a component of variance model. Its application appears promising, but the required assumptions should first be verified by an analysis of real data.

Under the model and assumptions expressed above, the distribution of peak primary loads, Y_{ij}, will have a normal distribution with a mean of $Y = \mu + B_i$ and a variance of $\sigma_Y^2 = \sigma_B^2 + \sigma_\epsilon^2$. The analysis of variance will provide estimates of σ_B^2 and σ_ϵ^2 with sufficient degrees of freedom to neglect sampling error of these parameters. The

estimate of B_i, however, will be based on only the number of records obtained for flight condition i. To be conservative, upper P percentage confidence limits can be placed on B_i by means of the formula $B_u = B_i + Z_p \sigma_B$, where Z_p is the Pth percentile of the standard normal distributions. The determination of $Y_u = \mu + B_u$ and σ_Y specifies the conservative distribution of peak loads expected to be encountered in flight condition i. Given the total number of such peaks as estimated from the mission spectra, the expected number in bands of load can easily be calculated as the desired input to a damage accumulation model. The illustrative example of the Appendix contains a numerical example of the use of this procedure.

Factors for Higher Order Cycles

Dynamic systems are subjected to cyclic loading at least once per rotor blade revolution. Generally, the number of significant load cycles per rotor revolution coincide with the number of blades. As mentioned earlier each critical point loading record by detailed flight condition is reviewed to ascertain the resonant oscillating load frequencies per revolution. If, for example, 3/rev cyclic loading occurs, these loads are seldom of equal amplitude and can be resolved into primary, secondary and tertiary load cycles per revolution. All previous discussion has dealt with the use of the primary cycles. Automatic processing allows the determination of load amplitude ratios for secondary to primary (X_i) and tertiary to primary (Y_i) for each rotor revolution. The information can be put into a format which is useful for merging or for individual comparisons. Each record will have X_i and Y_i values where i = 1 to n.

The mean (μ) for any grouping is estimated by

$$\mu = \Sigma X_i / n \tag{19}$$

In like manner the standard deviation (σ) is estimated by

$$\sigma = \left[\frac{1}{n-1} \Sigma X_i^{\,2} - \frac{(\Sigma X_i)^2}{n(n-1)} \right]^{1/2} \tag{20}$$

The standard deviation of the mean (σ_μ) is estimated as

$$\sigma_\mu = \frac{\sigma}{\sqrt{n}} \tag{21}$$

By assuming that the ratios are normal, at a confidence level of 97.7%, the mean will not be greater than $\mu + 2\sigma_{\mu}$. Values for $\mu + 2\sigma_{\mu}$ can thus be determined for the various cycle type and dynamic system.

These derived load amplitude ratios ($\mu + 2\sigma_{\mu}$) can thus be designated as X for secondary and Y for tertiary and used as a multiplying factor to the primary peak stress to assess their effect on damage.

SAFE-LIFE CALCULATION PROCEDURES

In the analytical review of the five manufacturers' procedures, all determined safe life based on "Miners Cumulative Damage Theory". This theory, simply stated, is

> All applied cycles of stress above an "endurance limit" produce fatigue damage. Furthermore, the fractional life reduction is calculable as the summation of ratios of applied cycles (n) to fatigue life (N) provided each ratio is obtained at the same cyclic and steady-state stress amplitudes. The fatigue life is defined as the amount of time required for the sum of these damage ratios to equal unity.

In addition to being simple to apply, this theory has also been found to be reasonably valid when the loading spectrum is random and does not contain extreme loads (i.e., loads which induce critical point stresses in the neighborhood of the yield stress).

The previous sections have discussed methods and factors to be considered in determining the necessary input for a rational fatigue life evaluation. Figure 7 illustrates a simplified overview of the development interface requirements in determining the fatigue life estimate of a helicopter. The starting point is block 1, the MISSION SPECTRUM, which may be based on initial design requirements or on recently obtained operational flight loads data. This mission spectrum includes percentage times assigned to the basic conditions as well as the required parametric splits to designate each detailed flight condition and its time apportionment. Derived basic condition percentages are shown in Table 3 for the various helicopter mission types. These percentages as well as number of parametric class interval assignments, mid-range values, and percentage probabilities should be negotiated between the Army and the manufacturer to ensure the consideration of all potential damaging combinations. The resultant products from these assignments are defined detail flight conditions and the expected number of rotor revolutions for each condition per 100 hours of helicopter operation.

Block 2 of Figure 7 in essence represents a multitude of operations and iterations necessary to arrive at a preliminary design. The scope of detailed effort required entails a schematic larger than Figure 7 itself, since it is necessary to design components and systems and to verify fatigue lives without the benefit of full-scale fatigue test and flight strain survey results. The manufacturer utilizes the design mission spectra, performance requirements, and past experience in conjunction with huge computer programs to

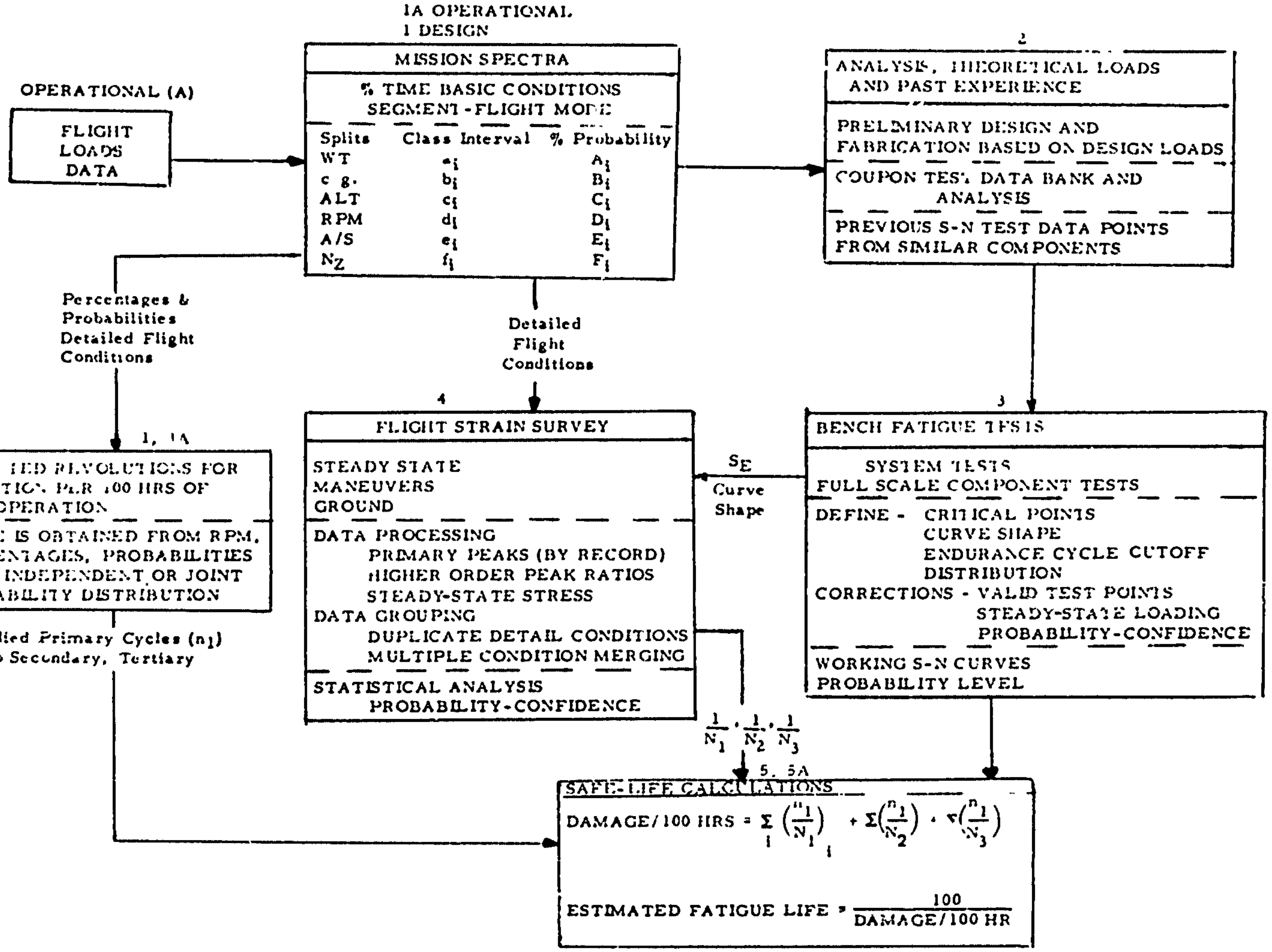

Figure 7. Schematic Damage and Estimated Life Calculations.

iteratively arrive at component and system designs. Large data banks of coupon tests and similar component full-scale fatigue tests are incorporated in the design procedure and are also used in conjunction with the analytical loads prediction programs to prepare for full-scale bench fatigue tests.

Block 3 points out the considerations required in arriving at full-scale working S-N curves. The full-scale system and component fatigue tests should be reasonably close to flight conditions especially with regard to load coupling and steady-state loading. Analytically determined design loading and previous experience will facilitate the defining of failure load type, choice of actuators, and instrumentation locations. Many of the critical points will be determined from system tests. The type of material and failure mode will define the curve shapes to be used. These curve shapes and the endurance cycle cutoff should be standardized for all manufacturers. It appears that the less conservative normal distribution should be assumed for defining the mean alternating stress and the standard deviation. Corrections should be applied to test data points where differences exist in the test steady-state load. These corrections should also be applied when differences exist between flight test steady-state loads and the value used for the S-N working curve. A methodology is presented in LABORATORY FATIGUE TEST METHODS for arriving at the proper working curves to give desired reliability with 90% confidence for various test sample sizes.

Block 4 shows the various processes used to designate loading distributions. It is in this block that the major differences exist between recommended treatment of data and the existing procedures. In fact the existing procedures between manufacturers show considerable variation in this area (see Existing Finalized Assessment). The recommended methodology is presented in the FLIGHT STRAIN SURVEY TECHNIQUES whereby primary peaks (one per rotor revolution) are processed to differentiate between damaging and nondamaging critical point-flight condition combinations. Duplicate tests are required for damaging or questionable damaging flight conditions. The results of these duplicate tests are used to generate statistical loading distributions in terms of percentage of occurrence vs percentage endurance limit. The damage boundary condition is defined as 100% endurance limit obtained from the proper critical point S-N working curves corrected to account for steady-state load differences. In this analytical approach the life cycles due to primary loads are determined directly from the statistical loading distribution with the aid of the appropriate S-N working curve and its related curve shape. That is, for each curve shape there is an expression for $N = g(S/S_E)$ where $100\ (S/S_E)$ equals the percentage endurance limit. In order to calculate the damage for any detailed flight condition - critical point combination, it is necessary to:

(1) Put loading distribution in histogram format (j intervals of % endurance limit vs % occurrence).

(2) Read mid-range percentage endurance limit and percentage occurrence.

(3) Solve for S/S_E = % endurance limit/100

(4) Solve for $N = g(S/S_E)$

(5) Solve for (% occurrence/100 N)$_i$

Repeat procedure for new class interval at Step 2 until all are accounted for (j intervals). NOTE: When $N_i >$ endurance number of cycles (e.g., 10^7), this ith increment is set to zero.

(6) The equivalent damage per revolution is

$$\sum_{i}^{j} \left(\frac{\%\ \text{occurrence}}{100N}\right)_i = \frac{1}{N_1} \text{ equivalent} \qquad (22)$$

(7) If secondary and tertiary cycles are found to be significant, multiply S/S_E by X and repeat Steps 2 through 6 to define $1/N_2$ equivalent. Repeat steps for tertiary loading using Y factor to determine $1/N_3$ equivalent.

Blocks 5 and 5A SAFE-LIFE CALCULATIONS show how the individual flight condition damages are accumulated to give a damage rate per 100 hours of helicopter operation. The necessary ingredients are the applied cycles (n) and the life cycles (N) in some form, such as shown in the Block 4 discussion. The applied cycles per 100 hours are determined by using the basic condition percentages, the splits and the appropriate RPM for the detailed condition. These values will probably vary between the design mission spectra and the spectra obtained from operational data. The number of revolutions per 100 hours of operation is expressed as:

$$n = (\%\ \text{basic condition}) \left(\frac{A_i}{100}\right) \left(\frac{B_i}{100}\right) \left(\frac{C_i}{100}\right) \left(\frac{D_i}{100}\right) \left(\frac{E_i}{100}\right) 60 \text{ RPM} \qquad (23)$$

This n constitutes the number of applied primary cycles (n_1) and also the same number of secondary, tertiary, etc., applied cycles. The damage by critical point for each detailed flight condition per 100 hours helicopter operation is thus

$$\frac{n_1}{N_1} + \frac{n_1}{N_2} + \frac{n_1}{N_3}, \text{ etc.}$$

The introduction of each new flight condition will give new damage increments which are accumulated to give the resultant critical point damage accrued in the designated 100 hours. Once all detailed flight conditions

have been considered, new critical points are introduced and the procedure is repeated. The estimated fatigue life for each critical point is thus 100 divided by the damage per 100 hours of operation.

CONCLUSIONS AND RECOMMENDATIONS

This report consists of the definition of a standardized methodology for the evaluation of fatigue lives for dynamic components of helicopters. The conclusions drawn in arriving at this definition are embodied in Tables 1, 7, and 9 and in Figure 7. However, in addition to these comments, which relate specifically to the standardized methodology, the following more general conclusions should be noted.

1. Continuing flight loads programs are necessary to monitor variations in operational usage for the vehicle types. Greater refinement is necessary in the collection and processing of flight loads data. This includes a greater number of mission segment - operating mode combinations as well as the parametric combinations of weight, center of gravity, airspeed, etc., which may affect the dynamic component loading. It is necessary to maintain liaison between the helicopter manufacturer and the flight loads contractor at least during the planning for flight loads programs.

2. A definite need exists to define a standardized mission spectra format which will provide the interface between the operational flight loads and the flight strain survey test conditions. A starting point for this standardization is provided by the basic condition and percentage time assignments of Table 3.

3. Basic conditions which define maneuvers will require the consideration of at least two additional parameters (e.g., load factor and stick rate). Correlation exists between damage sensitivity and these two parameters; thus, emphasis must be placed on defining realistic parametric combinations of weight, airspeed, etc., with load factor level or stick rate levels. Sorting operational data by the Table 3 format alleviates this problem.

4. In determining a working S-N curve from fatigue tests of specimens, the common practice of reducing the sample average endurance limit stress by three sample standard deviations may not be as conservative as is often inferred. It should be noted that the 3σ reduction of the average S-N curve does not give a 99.87% probability curve. The actual level of probability depends on the sample size and the selected degree of confidence.

5. The peak stresses observed during a short recording period of a flight strain survey may not be representative of the large stress cycles for a given detailed flight condition. The use of the maximum stress cycle observed in a short recording period may not be a conservative procedure.

Although the report defines a fatigue life methodology, there are elements of the method which require further study prior to implementation as a standardized method. In particular, the following are recommendations for future development and analysis.

1. Given the relationship between stress and N_Z level, real data should be analyzed from the point of view of correlating time at N_Z levels with frequency of occurrence of N_Z level.

2. Flight test survey data should be analyzed to evaluate the degree of validity of the statistical assumptions required by the recommended peak stress characterization method.

3. An analytical study should be performed (based on representative data) to evaluate the sensitivity of fatigue life predictions to variations in percentage time assignments and to the variations in the determination of the endurance limit stress.

4. A study should be performed whose objective is to arrive at specific mathematical models for S-N curves for the materials and test conditions of interest in the helicopter industry.

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APPENDIX A

ILLUSTRATIVE EXAMPLE

In order to illustrate the concepts of the recommended fatigue life methodology, a simple numerical example is presented in the following pages. First, a mathematical framework for damage calculations is presented which sets the stage for the example. Then follows a brief example of the major steps of the life evaluation of a helicopter component. Of necessity, the example is simple and all data are fictitious.

MATHEMATICAL FRAMEWORK FOR DAMAGE CALCULATIONS

Miner's damage computations consist of summations of ratios of number of induced (or anticipated) cycles of a particular stress to the number of cycles to failure of the structure at that stress. For a helicopter this summation can be conveniently modeled for damage/1000 hours by

$$D/1000\ \text{hr} = \sum_k \sum_i \sum_j D_{kij} \qquad \text{(A-1)}$$

where

D_{kij} = damage/1000 hr due to the k^{th} order stress peaks of a rotor revolution being at stress level j in the i^{th} detailed flight condition.

Now

$$D_{kij} = n_{kij}/N_{ij} \qquad \text{(A-2)}$$

where

N_{ij} = number of cycles to failure at mean stress of flight condition i and alternating stress level j

n_{kij} = number of k^{th} order cycles of stress level j in detailed flight condition i in 1000 hr

$$= (1000\ \text{hr})\ (\text{RPM})\ (60)\ (T_i)\ P_{ki}\ (S_j - \Delta < S < S_j + \Delta) \qquad \text{(A-3)}$$

and

$$T_i = (\text{Percentage of time in condition } i)/100.$$

$P_{ki}(S_j - \Delta<S<S_j + \Delta)$ = Probability that the k^{th} order stress peak of a rotor revolution in condition i is within Δ of S_j. Δ is chosen so that insignificant errors are introduced by saying all stresses within Δ of S_j are equal to S_j.

A schematic diagram for obtaining the required input for this calculation was presented in Figure 7. The blocks of this figure correlate with the above formulation in an obvious fashion. Block 1, 1A is concerned with defining the total number of detailed flight conditions and estimating T_i. Block 3 relates to the determination of N_{ij} for the component of interest. The recommended procedures for the flight strain survey of Block 4 will result in evaluating $P_{ki}(S_j - \Delta<S<S_j + \Delta)$. Finally, Block 5, 5A relates the combination of these data elements to arrive at a life estimate.

MISSION SPECTRA

Assume that the helicopter under consideration is in the observation class and that the anticipated basic condition flight time is defined by the percentages of Table 3. Further, assume that assigned percentage times in the detailed flight conditions are as expressed in the example of the subsection entitled Parametric Splits. The total number of detailed flight conditions would then be the sum of the number of parametric splits possible for each basic condition. If the basic condition is forward level flight and all parametric splits of the example are realizable for this basic condition, there would be a total of 270 detailed flight conditions.

Now, in order to limit the size of this example, the contribution to the damage/1000 hours of operational flight from only one of these conditions will be calculated. Let the detailed flight condition be defined as forward level flight (38.08%), $W \geq 0.9$ max design gross weight (25%), nominal c.g. (60%), RPM >0.98 design RPM (90%), alt<2000 ft (30%), $0.8 \leq V/V_H < 0.96$ (12%). For this set of detailed flight conditions, identified as Condition 1,

$$\begin{aligned} T_1 &= (38.08)(.25)(.60)(.90)(.30)(.12)/100 \\ &= 0.0018507 \qquad \text{(A-4)} \end{aligned}$$

Thus, 0.185% of this helicopter's flight time is spent at the detailed flight condition defined by all of the above conditions.

BENCH FATIGUE TESTS

Assume that the component under consideration is made from 7075-T6 aluminum and that smooth, rotating beam coupon data would be most representative for the component. For this combination the S-N curve shape parameters from Table 8 are $\alpha/\bar{E} = 76$ and $\gamma = 0.40$. Now assume that 6 specimens are tested to failure at a constant mean stress at 6 levels of alternating stress. Table A-1 presents typical data for the alternating stress levels, number of cycles to failure, and the corresponding calculated endurance limit stresses. The endurance limit stresses were calculated by means of Equation 9 which for the above curve shape values is given by

$$E_i = \frac{S_i}{1 + 76\ N^{-0.4}} \tag{A-5}$$

The sample mean and standard deviation for these endurance limit stresses are $\bar{E} = 5072$ and $\sigma_E = 162$. Assume that an endurance limit stress value is desired such that there is 90% confidence that the value will be exceeded by at least 99% of the population of endurance limit stresses. For these values of $P = 99$, $\beta = 90$ and sample size $n = 6$, Figure 6 yields a value of $K(99, 90, 6) = 4.2$. Thus, from Equation 10, there is 90% confidence that 99% of the endurance limit stresses will be greater than

$$\begin{aligned} S_E = E_{p,\beta} &= \bar{E} - K(99, 90, 6)\sigma_E \\ &= 5072 - 4.2(162) \\ &= 4392 \end{aligned} \tag{A-6}$$

The working S-N curve is determined by Equation 11 as

$$\begin{aligned} S_j &= S_E + S_E(\alpha/E_c)\ N_j^{-\gamma} \\ &= 4392 + 4392\ (76)\ N_j^{-0.4} \\ &= 4392 + 3.338\ (10^5)\ N_j^{-0.4} \end{aligned} \tag{A-7}$$

The number of cycles to failure for alternating stress level j is given by

$$N_j = \left(\frac{3.338 \times 10^5}{S_j - 4392}\right)^{2.5} \tag{A-8}$$

TABLE A-1 EXAMPLE S-N DATA FOR SPECIMEN FATIGUE TESTS

Alternating Stress	Cycles to Failure	Corresponding Endurance Limit Stress
9000	1.287×10^5	5335
8500	1.131×10^5	4932
8000	1.567×10^5	4893
7500	3.455×10^5	5127
7000	5.073×10^5	5011
6500	1.373×10^6	5132

FLIGHT STRAIN SURVEY

The technique proposed for analyzing flight strain survey data as input to a fatigue analysis consists of estimating the distributions of peak load cycles in each detailed flight condition for the primary, secondary, etc., cycles of a rotor revolution. The method depends upon the assumption of normality (which needs to be investigated) and accounts for two sources of variation that affect the magnitude of stress peaks in a sample. The first of these is the peak-to-peak variation within a given sample of data and is denoted by σ_ϵ^2. The second is the variation that is the result of non-repeatability of flight conditions for different excursions into the "same" detailed flight condition. These sources of variability were modeled by means of the equation

$$Y_{ij} = \mu + B_i + \epsilon_{ij} \qquad \text{(A-9)}$$

where μ is an overall mean peak primary load, B_i is a random differential effect due to different excursions into the flight condition, and ϵ_{ij} is a random differential effect due to peak-to-peak variations within a single excursion into the flight condition.

An analysis of variance of the peak data from many data samples from many different flight conditions will provide estimates of σ_B^2 and σ_ϵ^2. It is beyond the scope of this example to assume data for this number of data samples. Therefore assume that this analysis has been performed and that σ_ϵ = 500 psi and σ_B = 100 psi. If on the two excursions into the detailed flight condition of the example, the average of the primary peaks for the excursions were 3600 and 3800 psi, then the mean peak stress for the flight condition is estimated by 1/2 (3600 + 3800) = 3700 psi. In order to insure that this mean stress level is not low due to sampling error, the estimate of the mean stress is increased by the proper multiple of σ_B to achieve the desired level of confidence. If we want to be 99% confident that the true mean peak stress for the flight condition lies below the value to be used in the analysis then Z_p = 2.33. Thus, for this degree of conservativeness the mean peak stress for this detailed flight condition is taken to be

$$\begin{aligned} \overline{Y}_\mu &= \mu + B_i + Z_p\sigma_B \\ &= 3700 + (2.33)(100) \\ &= 3933 \qquad \text{(A-10)} \end{aligned}$$

The standard deviation of the peak stresses is

$$\sigma_y = \left[\sigma_\epsilon^2 + \sigma_B^2\right]^{1/2}$$

$$= \left[(500)^2 + (100)^2\right]^{1/2}$$

$$= 510 \text{ psi} \qquad \text{(A-11)}$$

Thus, the working distribution of loads as determined from the flight strain survey data for the detailed flight condition have a normal distribution with a mean of 3933 psi and a standard deviation of 510 psi. (For purposes of this example, assume that the higher order stress cycles of a rotor revolution have been shown to produce stresses that do not exceed the endurance limit.)

Now assume that all stresses within 50 psi of a given stress produce equivalent damage; i.e., Δ of Equation A-3 is 50 psi. Then, the percentage of peaks, $P_{ki}(S_j - \Delta < S < S_j + \Delta)$, for 100 psi increments for the stress ranges of interest ($S > 4400$ psi = working endurance limit) can be calculated as in Table A-2. In this table S is the midpoint of the stress interval. $S \pm \Delta$ are the endpoints of each interval, $Z = S \pm \Delta - Y_\mu / \sigma_y$ are the standardized endpoints, $\Phi(Z)$ is the cumulative normal probability tabled in standard statistical texts, and P(S) is the percentage of stress peaks in the range $S \pm \Delta$ which are assumed to lie at the midpoint value of S.

DAMAGE CALCULATION

All the parameter values required in the calculation of damage for the flight condition have now been estimated. According to the assumptions made in the sample problem, only the first order stress cycles of a rotor revolution in this detailed flight condition can cause damage and only the damage/1000 hr of detailed flight condition 1 is being considered. Thus, the damage calculation for detailed flight condition 1 (assuming that RPM = 300) is given by

$$D_{11}/1000 \text{ hr} = \sum_{j=1} n_{11j}/N_{1j} \qquad \text{(A-12)}$$

where

$$N_{1j} = \left(\frac{3.338 \times 10^5}{S_j - 4392}\right)^{2.5}$$

$$n_{11j} = (1000 \text{ hr})\,(300 \text{ RPM})\,(60)\,(T_1)P_{11}(S_j - 50 < S_j < S_j + 50)$$

TABLE A-2 EXAMPLE CALCULATION FOR PERCENTAGE OF PEAKS IN STRESS RANGES

S	S±Δ	Z	Φ(Z)	P(S)
	4400	0.92	.8212	
4450				.0453
	4500	1.11	.8665	
4550				.0384
	4600	1.31	.9049	
4650				.0283
	4700	1.50	.9332	
4750				.0222
	4800	1.70	.9554	
4850				.0159
	4900	1.90	.9713	
4950				.0104
	5000	2.09	.9817	
5050				.0073
	5100	2.29	.9890	
5150				.0044
	5200	2.48	.9934	
5250				.0029
	5300	2.68	.9963	
5350				.0017
	5400	2.88	.9980	
5450				.0009
	5500	3.07	.9989	
5550				.0006
	5600	3.27	.9995	
5650				.0005

and

$$T_1 = 0.0018507$$

$$P_{11j} = \text{values from Table A-2}$$

The individual ratios and their sums are shown in Table A-3 for the sample problem. The total damage/1000 hours for this detailed flight condition is estimated to be 30.851×10^{-5}.

TABLE A-3 EXAMPLE CALCULATION FOR DAMAGE/1000 HR IN A DETAILED FLIGHT CONDITION

S_j	P_{11j}	n_{11j}	N_{1j}	n_{11j}/N_{1j}
4450	.0453	1509	25.127×10^8	$.060 \times 10^{-5}$
4550	.0384	1279	2.05×10^8	$.623 \times 10^{-5}$
4650	.0283	943	6.021×10^7	1.566×10^{-5}
4750	.0222	740	2.655×10^7	2.788×10^{-5}
4850	.0159	530	1.434×10^7	3.696×10^{-5}
4950	.0104	346	8.752×10^6	3.953×10^{-5}
5050	.0073	243	5.796×10^6	4.192×10^{-5}
5150	.0044	147	4.070×10^6	3.612×10^{-5}
5250	.0029	97	2.985×10^6	3.249×10^{-5}
5350	.0017	57	2.266×10^6	2.515×10^{-5}
5450	.0009	30	1.768×10^6	1.697×10^{-5}
5550	.0006	20	1.411×10^6	1.418×10^{-5}
5650	.0005	17	1.147×10^6	1.482×10^{-5}
			D_{11}/1000 hours =	30.851×10^{-5}

LIST OF SYMBOLS

a_i	number of weight class intervals
A_i	percent probability in a_i weight range
ALT	altitude
A/S	airspeed expressed in $\%V_H$ or $\% V_{NE}$
b_i	number of c.g. class intervals
B_i	percent probability in b_i c.g. range, random differential effect between duplicate flight records
c_i	number of RPM class intervals
C_i	percent probability in c_i RPM range
CCF	cycle count factor
c.g.	center of gravity
Dam.	damage
d_i	number of altitude class intervals
D_i	percent probability in d_i altitude range
e_i	number of airspeed class intervals
E_i	percent probability in e_i airspeed range
E	endurance limit (individual data point)
$\bar{E}$	mean endurance limit (full-scale tests)
$\bar{E}_c$	mean endurance limit (coupon tests)
E_p	Pth percentile of endurance limit stress distribution
$E_{p,\gamma}$	tolerance limit of endurance limit stress distributions
f_i	number of N_Z class intervals

LIST OF SYMBOLS (Continued)

F_i	percent probability in f_i range for N_Z
FM	frequency modulation
IGE	in ground effect
$K(P, \beta, n)$	factor function of (probability level, confidence level, data points)
n	applied cycles, number of data points
n_e	equivalent applied cycles
N	life cycles or allowable cycles
N_e	equivalent life cycles
N_Z	load factor in g's
P	designated probability level
R	ratio of S_{min}/S_{max}
RPM	rotor revolutions per minute
S, S_{alt}	alternating stress amplitude
S_E	endurance limit of working S-N curve
S_{max}	maximum stress
S_{mean}	mean or steady-state stress
S_{min}	minimum stress
S_{ult}	ultimate tensile stress of material
T.O.	takeoff
V_H	limit airspeed in level flight
V_{NE}	not to exceed airspeed limitation

LIST OF SYMBOLS (Continued)

WT	gross weight
X_i	ratio of secondary to primary alternating stress
X	factor for determining secondary cycle damage
Y_i	ratio of tertiary to primary alternating stress
Y	factor for determining tertiary cycle damage
Y_{ij}	load distribution model
Z_P	the Pth percentile of the standard normal distribution
α	parameter which defines S-N curve shape
β	designated level of confidence
γ	parameter which defines S-N curve shape
ϵ_{ij}	random differential effect of primary peak in the same flight record
μ	mean value, advance ratio $V/\Omega R$
σ	standard deviation
σ_μ	standard deviation of the mean
σ_β^2	variance of peak cycles between duplicate recordings
σ_ϵ^2	variance of peak cycles within a recording
σ_y^2	variance of the load distribution model
ΩR	rotor tip speed, ft/sec